DiCoRD 2026 Abstracts

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Dawes, Jonathan, *Department of Mathematical Sciences, University of Bath*
Minimal reaction schemes for pattern formation

Pattern formation is often characterised in terms of reaction schemes involving small numbers of reagents playing specific roles, for example the activator-inhibitor combination. Recently we have establishing the complete catalogue of minimal reaction schemes, containing only elementary reactions, that allow a pattern-forming instability to occur.

From the 31 elementary stochastic chemical reactions involving at most two reactants it turns out that there are only 25 minimal combinations (out of around 50,000 possible combinations) that allow a pattern-forming instability to arise in the related mean-field PDEs. Eleven of these involve only three reactions; these all generate patterns in which the reactants fluctuate in-phase (which we refer to as 'type I'). Four reactions are required to generate anti-phase patterns ('type II') and there are 14 such minimal reaction schemes.

The minimal reaction schemes allow us to separate out the different roles played by stoichiometry and by reaction rates. Further, for all 11 reaction schemes of type I, the linear behaviour can be summarised on a single regime diagram. Overall, the catalogue provides new minimal examples through which we can learn more about the relation between individual-based and PDE-based models. This is joint work with Fraser Waters and Christian (Kit) Yates (University of Bath).

References:
F.R. Waters, C.A. Yates and J.H.P. Dawes, Physica D 471, 134427 (2025).
F.R. Waters, C.A. Yates and J.H.P. Dawes, J. Roy. Soc. Interface 21:20230490 (2024).

Alan Turing's later work on morphogenesis

Turing's well-known 1952 publication (his only published work in mathematical biology) covers only a small part of his perceptive thinking on morphogenesis. Unpublished archive material reveals the fascinating genesis of much more complex ideas that were re-discovered only in the 1970s. I will discuss the historical scientific context to the 1952 paper and its initial reception, and what can be learned from the archive material about his research programme in his final years. This later, unpublished, work included a nonlinear and nonlocal extension of the model fourth-order PDE that is generally known as the Swift-Hohenberg equation: Swift and Hohenberg, in 1977, were unaware that Turing had written it down roughly 24 years earlier.

Reference:
J.H.P. Dawes, Historia Mathematica 43, 49-64 (2016).

Diez, Antoine, *RIKEN iTHEMS*
Self-organized mechanochemical instabilities drive the emergence of digit tissue morphogenesis

How complex anatomical structures like digits emerge from unstructured tissues remains a fundamental question in developmental biology. While Turing-type reaction-diffusion models explain the periodic pre-patterning of digits, the physical principles driving 3D morphogenesis remain incompletely understood. We develop a limb-mesenchymal organoid system that spontaneously forms elongated, digit-like protrusions, and use cell-based computer simulations to identify sufficient microscopic mechanisms reproducing the experimental morphogenesis We find that symmetry-breaking and elongation of digits arise from a combination of differential cell adhesion and morphogen-induced chemotaxis and convergent-extension. Coarse-graining the agent-based model yields a continuum description as a Cahn-Hilliard-type system, which describe ≪ fingering instabilities ≫ in fluid physics. Our findings suggest that vertebrate digit formation is driven by a mechanical fingering instability acting in concert with chemical patterning.

Fukao, Takeshi, *Faculty of Advanced Science and Technology, Ryukoku University*
Robin-type transmission problems and aspects of dynamic boundary conditions

In this talk, we consider a Robin-type transmission problem for the Allen–Cahn equation. It is rigorously proved that the zero limit and the infinity limit of the permeability parameter lead to convergence to a problem in which two Allen–Cahn equations become independent and a problem in which two equations merge into one, respectively. Based on the interesting fact that the transmission problem converges to a problem with dynamic boundary conditions through the zero-thickness limit, it turns out that these parameter limit problems are closely related to the convergence from Robin-type boundary conditions to two types of problems known as the GMS model and the LW model in the Cahn–Hilliard equation with dynamic boundary conditions, which have been actively studied in recent years. We observe that the patterns on the boundary described by these problems exhibit differences in the manner of gradation, arising from the differences in how dynamic boundary conditions enter the system.

Ishii, Hiroshi, *Research Institute for Electronic Science, Hokkaido University*
Spot Solutions to a Neural Field Equation on Curved Surfaces

This talk considers the Amari model, an integro-differential equation that serves as one of the fundamental neural field models. Neural field equations are used to describe the temporal evolution of activity patterns in neural tissue and to understand the collective dynamics of neuronal populations at the mesoscopic level.

In recent years, increasing attention has been paid to the influence of geometry-particularly that of the brain-on pattern formation, as well as to the constraints that curved surfaces impose on solution behavior. To investigate these geometric effects, we study both the sphere and a slightly deformed sphere (spheroid) as model surfaces, and analyze how the structure and stability of spot solutions depend on surface geometry.

First, we construct spot solutions on the sphere and characterize their linear stability via spectral analysis of the linearized operator. Next, we perform a perturbation analysis from the sphere to the spheroid, constructing spot solutions localized near the north pole and deriving the eigenvalues governing their stability. Finally, we carry out numerical simulations to validate the theoretical results and explore the dynamics of these solutions. Time permitting, we will also discuss recent related developments.

This work is based on joint research with Riku Watanabe (Hokkaido University, D1).

Ishihara, Shuji, *Graduate School of Arts and Sciences, University of Tokyo*

Pattern dynamics driven by curved surfaces

Pattern formation on curved surfaces-such as spheres and tori-has attracted substantial attention since the seminal work of A. M. Turing. Most previous studies have assumed that Turing patterns on curved surfaces are stationary, as on a flat plane. In the present talk, we show that Turing patterns can spontaneously start to propagate and even exhibits complex motion like chaos on general curved surfaces[2,3]. We mainly study reaction-diffusion systems on axisymmetric surfaces, using parameter sets that yield stationary patterns on a flat plane. This setup allows a systematic analysis of admissible solutions, including modes that travel in the azimuthal direction. We further demonstrate that the onset of propagation depends on the symmetries of both the surface geometry and the emerging pattern. Our results identify a novel and generic curvature-induced mechanism for pattern propagation, which has no counterpart in one-dimensional systems, and provide new insight into how surface geometry can control pattern dynamics.

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[2] R. Nishide and S. Ishihara, Phys. Rev. Lett. (2022).

[3] R. Nishide and S. Ishihara, Phys. Rev. E (2025).

Kato, Yuzuru, *Future University Hakodate*
Turing instability in quantum activator-inhibitor systems

Turing instability is a fundamental mechanism of nonequilibrium self-organization, yet it has thus far been investigated mostly in classical systems. In this study, we show that Turing instability can occur in a quantum dissipative system and analyze its quantum features, such as entanglement and the effect of quantum measurement. We propose a degenerate parametric oscillator with nonlinear damping in quantum optics as a quantum activator-inhibitor unit and demonstrate that a system of two activator-inhibitor units can undergo Turing instability when diffusively coupled with each other. The Turing instability induces nonuniformity and entanglement between the two units and gives rise to a pair of nonuniform states that are mixed due to quantum noise. Moreover, performing continuous measurement on the coupled system reveals the nonuniformity caused by the Turing instability. Our results extend the universality of the Turing mechanism to the quantum realm and provide a novel perspective on the possibility of quantum nonequilibrium self-organization.

Muolo, Riccardo, *RIKEN iTHEMS*
Cross-dimensional Turing patterns on higher-order networks

Pattern formation is a key feature of many natural and engineered systems, ranging from ecosystems to fluid dynamics and neural dynamics, just to mention a few. Turing instability provides one of the most famous theories for pattern formation in a continuous domain. On networks, Turing patterns were extended by Nakao and Mikhailov, allowing to model systems where the topology is intrinsically discrete, and, dynamical variables (species) are defined on the nodes, i.e., they interact in the nodes and flow among nodes by using network links. Currently, growing interest is addressing problems related to the formation of Turing patterns of species located on the nodes of simple hyperbolic and higher-order networks.

However, in a number of real systems, including the brain and the climate, dynamical variables are not only defined on nodes but also on links, triangles and higher-dimensional simplices, leading to topological signals.

The discrete topological Dirac operator is emerging as the key operator that allows cross-talk between signals defined on simplices of different dimensions, for instance among nodes and links signals of a network.

Recently, it was demonstrated that Turing patterns can be formed as well in this topological setting in which nodes, links and higher-order building blocks sustain different topological signals and are coupled by the Dirac operator. For instance this mechanism leads to pattern formation not only on nodes but also on links of networks.

Here, we propose a mathematical framework able to generate dynamical Turing patterns of topological signals defined on nodes and links of networks.

In particular, we consider two species located on the nodes of the network and one species located on the links; let us observe that the complementary setting has been considered as well. In order to couple these different signals, we define a three-way Dirac operator and its associated gamma matrix. This operator allows the considered 3 species of topological signals to interact and cross-talk. Indeed, the Hodge-Dirac operator couples signals on 0-simplices with the signals on 1-simplices by projecting signals one dimension up or one dimension down. Moreover, the gamma matrix associated to the Hodge-Dirac operator allows to compress the projected two-dimensional signals of the nodes into a one dimensional projected signal of the links and, vice versa, expand the one-dimensional signal of the links into a two-dimensional projected signal on the nodes.

This framework accounts for dynamical Turing patterns with a very rich dynamical behavior even without the (Hodge-Laplacian) diffusion term, i.e., occurring solely due to the Dirac operator."

Nishiura, Yasumasa, *Research Institute for Electronic Science, Hokkaido University*
Pattern Dynamics from Non-normal and Non-commutative Perspectives

A classical and widely cited example of an instability mechanism is the transition from laminar to turbulent motion in Reynolds' pipe flow. Although the eigenvalues of the linearized operator about the laminar profile indicate spectral stability, the laminar state is extremely sensitive to finite-amplitude disturbances and is not sustained in experiments at the corresponding parameter values. Considerable progress has been made in understanding the laminar-turbulent boundary, often described in terms of an edge state (or, more generally, a basin boundary in state space). In parallel, transient amplification mechanisms arising from the non-normality of the Navier-Stokes operator have been extensively studied: various shear-induced processes have been proposed and analyzed from both physical and mathematical viewpoints.

In this lecture, we aim to revisit pattern dynamics in reaction-diffusion systems through the lens of non-normality and non-commutativity, highlighting how these structural features can organize transient growth and shape dynamical transitions.

Ozawa Ayumi, *Center for Mathematical Science and Advanced Technology, JAMSTEC*
Phase reduction of reaction-diffusion systems with delay

Partial differential equations with delay often exhibit spatiotemporal oscillation. While the bifurcation theory unveils the condition under which such oscillatory patterns arise, it remains difficult to analyze how the oscillatory patterns respond to perturbations such as external forcing and interaction with other systems. To fill this gap, we aim to formulate phase reduction methods [1] for partial differential equations with delay. Specifically, in this talk, we reduce reaction-diffusion equations with discrete delays into phase equations that describe the modulation of the rhythm by perturbation [2]. As an example, we consider the Schnakenberg system with a discrete delay in one spatial dimension [3]. The theory enables us to obtain the phase sensitivity function, which quantifies how the rhythm of the oscillatory pattern is modulated depending on where and when the perturbation is applied. We also demonstrate that a variety of mathematical tools for phase equations, including an optimization method for mutual synchronization [4], can be immediately applied once the reduction is conducted. Thus, our theory enhances the analysis and control of oscillatory patterns induced by delay.

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Rademacher, Jens *University of Hamburg* **Fronts in some bistable multi-component reaction diffusion systems**

Fronts as one-dimensional interfaces connecting asymptotically constant states are manifestations of nonlinear effects and are highly relevant in applications such as phase transitions. Their study in reaction-diffusion equations and two-component systems has a long history, in particular in the bistable case of the Allen-Cahn phase field model with cubic nonlinearity. Much less is known in case of more components. A notable exception is a well-studied three component system of an Allen-Cahn equation coupled to two linear fields that can in particular act as refractory variables. With N such variables, yielding an N+1 component system, such systems turn out to be still amenable to rigorous analysis. Here some bifurcation results for fronts in such systems with a certain scale separation are discussed. In particular, these results imply non-trivial motion of front-like solution that can even be chaotic.

This is joint work with Martina Chirilus-Bruckner (Leiden) and Peter van Heijster (Wageningen), building on earlier work with additionally Hideo Ikeda (Toyama) and Arjen Doelman (Leiden)

Sato, Makoto, *Institute for Frontier Science Initiative, Kanazawa University*
Tiling mechanisms in the compound eye: From periodic hexagons to aperiodic monotiles

Tiling patterns are ubiquitous in biological systems, from insect compound eyes to columnar organization in the brain. Among them, hexagonal tiling is dominant, likely due to its favorable physical properties such as structural stability, minimal boundary length, and efficient space filling. The compound eye of the fruit fly consists of regularly arranged units forming a hexagonal pattern and provides an ideal model for studying tiling mechanisms. Interestingly, tetragonal patterns can also emerge in certain mutant conditions. Combining biological experiments and mathematical modeling, we propose that mechanical stretching of the tissue and concentric cellular growth cooperatively determine these tiling patterns.

When identical tiles are arranged with translational symmetry, the pattern is periodic, as seen in the compound eye. In contrast, an Einstein tile is a single shape that tiles the plane non-periodically, a problem that remained open for over 50 years until its discovery in 2023.

Interestingly, Einstein tiles share geometric features with hexagonal tilings and with cellular arrangements observed in biological tissues. We ask whether similar geometric and physical principles may underlie both biological tiling and aperiodic tilings, and discuss potential connections between them.

Suematsu Nobuhiko J., *Meiji University*

Macro- and Micro-Structures Generated by Diffusion-Induced Precipitation Reaction

The Liesegang pattern is a well-known discrete pattern produced by diffusion and chemical reactions. The typical setup for the Liesegang pattern is that one reactant is homogeneously distributed in a gel, and the other is dissolved in an aqueous solution placed on the gel. Although diffusion of reactants and precipitation reactions occur continuously, the products appear at discrete positions. In this study, we used a Hele-Shaw cell rather than a gel to prevent macroscopic flow. We focused on the general precipitation reaction between CrO_4^{2-} and Cu^{2+} and prepared aqueous solutions on both sides of the Hele-Shaw cell. The precipitates, $CuCrO_4$, formed discretely near the solution interface, and their crystal size changed periodically. The period of the discrete pattern decreased as reactant concentrations increased. The pattern formation was explained using a mathematical model that employed a step function to describe the precipitation reaction. Although the step function reproduces the precipitation process well, the saturation concentration must be specified artificially, and the dissolution process was neglected. In other words, the saturation concentration is determined by competition between dissolution and precipitation. Here, we proposed a mathematical model that did not use a step function and accounted for both dissolution and precipitation processes. Our model also reproduced experimental observations well. Recent experiments on Liesegang patterns have expanded to a variety of chemicals and precipitation formations, not only ion reactions but also nanoparticle aggregation, polymerization, and self-assembly processes. To explain those systems, mathematical models based on fundamental processes, namely precipitation and dissolution, are important. We believe that our suggested model can adapt to a wide range of materials systems.

Sugimoto, Takanori, *Department of Electrical, Electronic, and Information Engineering, Kansai University*
Hyperuniform-Multifractal transition in Quasicrystalline Bose-Hubbard model

Quasicrystals possess long-range order without translational symmetry, creating unique landscapes for quantum phenomena. Distinguishing their physical properties from those of periodic crystals remains a fundamental challenge. Recently, we investigate the Bose-Hubbard model on Penrose and Ammann-Beenker tilings using two distinct structural descriptors: hyperuniformity, which characterizes systems with suppressed large-scale density fluctuations, and multifractality, which describes complex, scale-invariant spatial distributions [1,2].

Using mean-field calculations, we determine the real-space distributions of the local superfluid amplitude and boson density. In both Mott insulating and superfluid phases, these distributions are found to be hyperuniform. Notably, the order metric-a measure of spatial pattern complexity-shows a significant enhancement at the phase boundary in quasicrystals, a feature absent in periodic square lattices. This suggests that hyperuniformity is a robust concept for differentiating crystalline and quasicrystalline bosonic systems.

Furthermore, we introduce random potentials to explore the Bose glass phase. We find that this phase exhibits multifractality, in contrast to the hyperuniform nature of the Mott and superfluid phases. Since similar multifractality occurs in periodic lattices, our results indicate that multifractality is a universal signature of the Bose glass phase, independent of the underlying system periodicity.

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Tanaka, Yoshitaro, *Future University Hakodate*
Diffusion-driven instability and critical cell size in a spatially discretized reaction-diffusion system on a lattice

In this talk, we analyze the conditions for diffusion-driven instability in reaction-diffusion systems spatially discretized on a lattice. Within the finite-volume method framework, we propose a discretized reaction-diffusion system and investigate the associated eigenvalue problem near its spatially homogeneous steady states. For a uniform one-dimensional lattice, we theoretically demonstrate the existence of a critical lattice size. When the lattice size is smaller than this critical value, the conditions for diffusion-driven instability coincide with those of the corresponding continuous reaction-diffusion system. Conversely, when the lattice size exceeds the critical value, the instability conditions exhibit characteristics unique to the discrete system. Moreover, we elucidate the modes of instability near the bifurcation point, showing in particular that, in cases of diffusion-driven instability inherent to the discrete model, the eigenmodes always form alternating patterns, independent of boundary conditions or the ratio of diffusion coefficients. Finally, we extend the analysis to nonuniform lattices and rigorously derive the conditions for diffusion-driven instability and the critical cell size in the specific case where the lattice exhibits a two-period structure.

Tomiyasu, Ryoko, *Institute of Mathematics for Industry (IMI), Kyushu University*
Discrete Geometric Packings Arising from Phyllotaxis and Its Connection to Pattern-Forming PDEs

Packing problems arise in various contexts ranging from number theory to biological pattern formation. Phyllotactic patterns generated by the golden-angle method exhibit quasi-periodic structures with high packing efficiency despite its simple generative rules. In [1], we extended the classical framework beyond rotationally symmetric surfaces to general n-dimensional manifolds endowed with orthogonal n-frame fields, and obtained a new method for generating uniform point configurations on the manifolds together with closed-form lower bounds for locally defined packing density.

This framework provides a unified treatment of periodic and non-periodic packings and yields new applications of algebraic lattice theory to uniform point sets and mesh generation algorithms.

From the modern perspective, it is natural to ask whether the connections between phyllotaxis and pattern-forming PDEs suggested in the past studies extend to the generalized phyllotactic patterns. As a possible direction, we point out that the local packing density function U defined on the manifold exhibits structures compatible with stationary solutions of reaction-diffusion-type equations. This observation suggests that continuum pattern-forming structures may arise naturally from discrete phyllotactic patterns.

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Watanabe, Takeshi, *Faculty of Information Science and Collaborative Innovation, Nagano University*
Bifurcation analysis of 2D morphologies in block copolymer nanoparticles via coupled Cahn-Hilliard equations

We investigate the two-dimensional morphologies of self-assembled block copolymer nanoparticles using coupled Cahn-Hilliard equations. These morphologies correspond to the critical points of the free energy functional, with the Morse index determining their linear stability. Generically, the landscape exhibits a vast set of minimizers, and the asymptotic behavior of any given trajectory is highly sensitive to both initial conditions and system parameters. Our primary goal is to identify the key saddle points that delineate the basins of attraction for these minimizers, thereby mapping the underlying free energy landscape. A network of saddle connections effectively guides the system's orbit toward a targeted minimizer. Concurrently, understanding how this landscape evolves with parameters is of critical importance, as controlling experimental variables—such as temperature, density, and composition—is essential for synthesizing the desired morphology.

Weyer, Henrik, *Kavli Institute for Theoretical Physics, UC Santa Barbara*
Coarsening, wavelength selection, and effective interfacial tension in mass-conserving reaction-diffusion systems

Intracellular processes must be precisely organized in space and time. A paradigmatic example is the symmetric division of bacteria, which, in E. coli, is orchestrated by the ATP-driven oscillation of Min proteins between the cell poles. Remarkably, two proteins of the Min system are sufficient for this pattern-formation process and also form a kaleidoscope of reaction-diffusion patterns in vitro. We will discuss conceptual models for protein pattern formation and see how mass conservation of the protein species can be used to construct fully nonlinear patterns, as well as predict their generic long-time dynamics independently of the specific mathematical form of the reaction term. We thereby uncover similarities of the reaction-diffusion patterns with phase-separating liquid mixtures and foams. Because these patterns are solely created by the reaction-diffusion process, we denote them as Turing mixtures and foams.

Yoshinaga, Natsuhiko, *Future University Hakodate*
Bayesian Model Selection of PDEs for Pattern Formation

Partial differential equations (PDEs) have been widely used to reproduce patterns in nature and to give insight into the mechanisms underlying pattern formation. Although many PDE models have been proposed, they rely on prior knowledge of physical laws and symmetries, and developing a model that reproduces a given desired pattern remains difficult. We propose a method to estimate the optimal dynamical PDE for a single snapshot of a target pattern in the stationary state, without ground truth. We apply our method to nontrivial patterns, such as quasi-crystals (QCs), a double gyroid and Frank-Kasper structures. By using the estimated parameters for the approximant of QCs, we successfully generate three-dimensional dodecagonal QCs from a PDE model. Our method works for noisy patterns, and the pattern is synthesised without the ground-truth parameters, which are required for the application to experimental data.