"Convergence, concentration and critical mass phenomena for a model of cell migration with signal production on the boundary"

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We consider a model of cell migration with signal production on the boundary. It consists in a diffusion equation with nonlinear nonlocal advection, complemented by a no-flux condition ensuring mass conservation.

For nonlinearities with polynomial growth, we first develop a local existence-uniqueness theory in optimal L^p spaces. With help of this tool, we next obtain the following results on the global behavior of solution:

- For small initial data, we have exponential convergence towards a constant.
- If, and only if, the growth of the nonlinearity is at least quadratic, we have concentration, i.e. finite time blowup, for large initial data.
- In the critical case of a quadratic nonlinearity, we observe a critical mass phenomenon *in any space dimension* (denoting by M the mass of the L^1 initial data):
  - for $M \leq 1$, the solution is global and bounded;
  - for $M > 1$, there exist initial data leading to finite time concentration.

This critical mass phenomenon is reminiscent of the well-known situation for the 2D Keller-Segel system. The global existence proof is delicate, based on a control of the solution by means of an entropy functional, via an ε-regularity type result.

- Finally we give some partial results on the localization and final profile of the boundary concentration and on the blowup rate.