ICMMA2014, Crowd Dynamics
in Meiji University

Jam formation and collective motions of self-driven particles

- Dynamics of dissipative system with asymmetric interaction -

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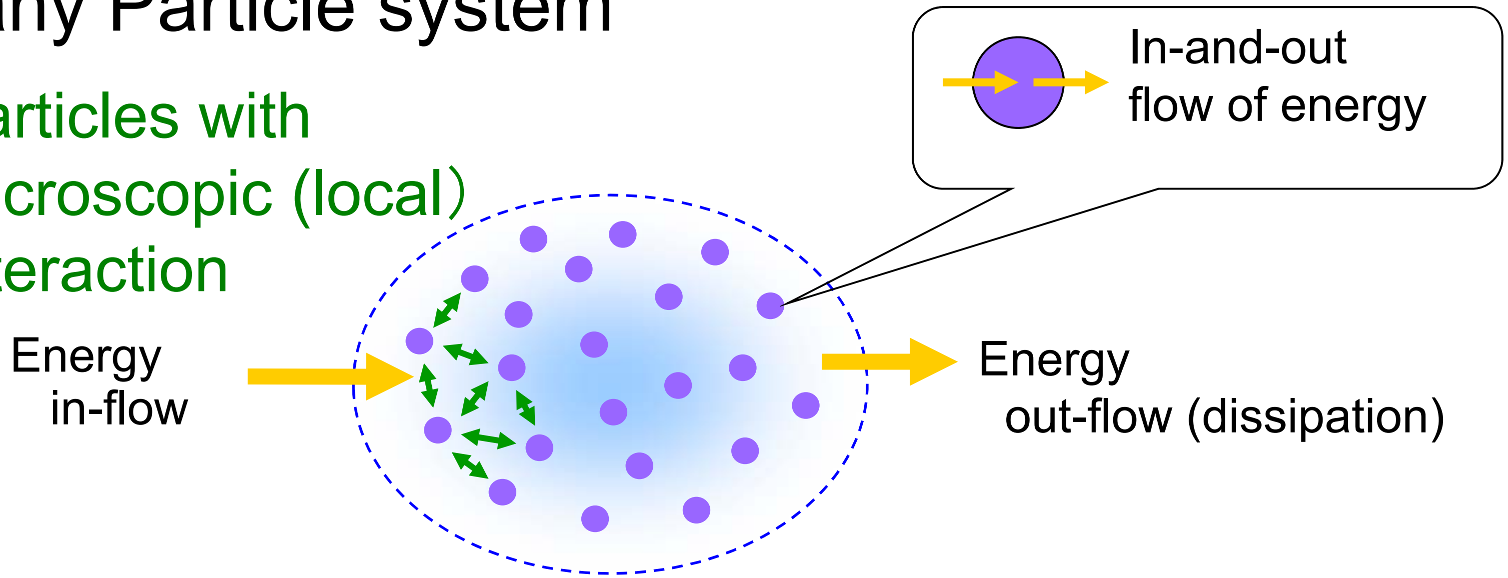
Self-driven particles as non-equilibrium dissipative many Particle system

Particles with microscopic (local) interaction



Emergence of macroscopic phenomena

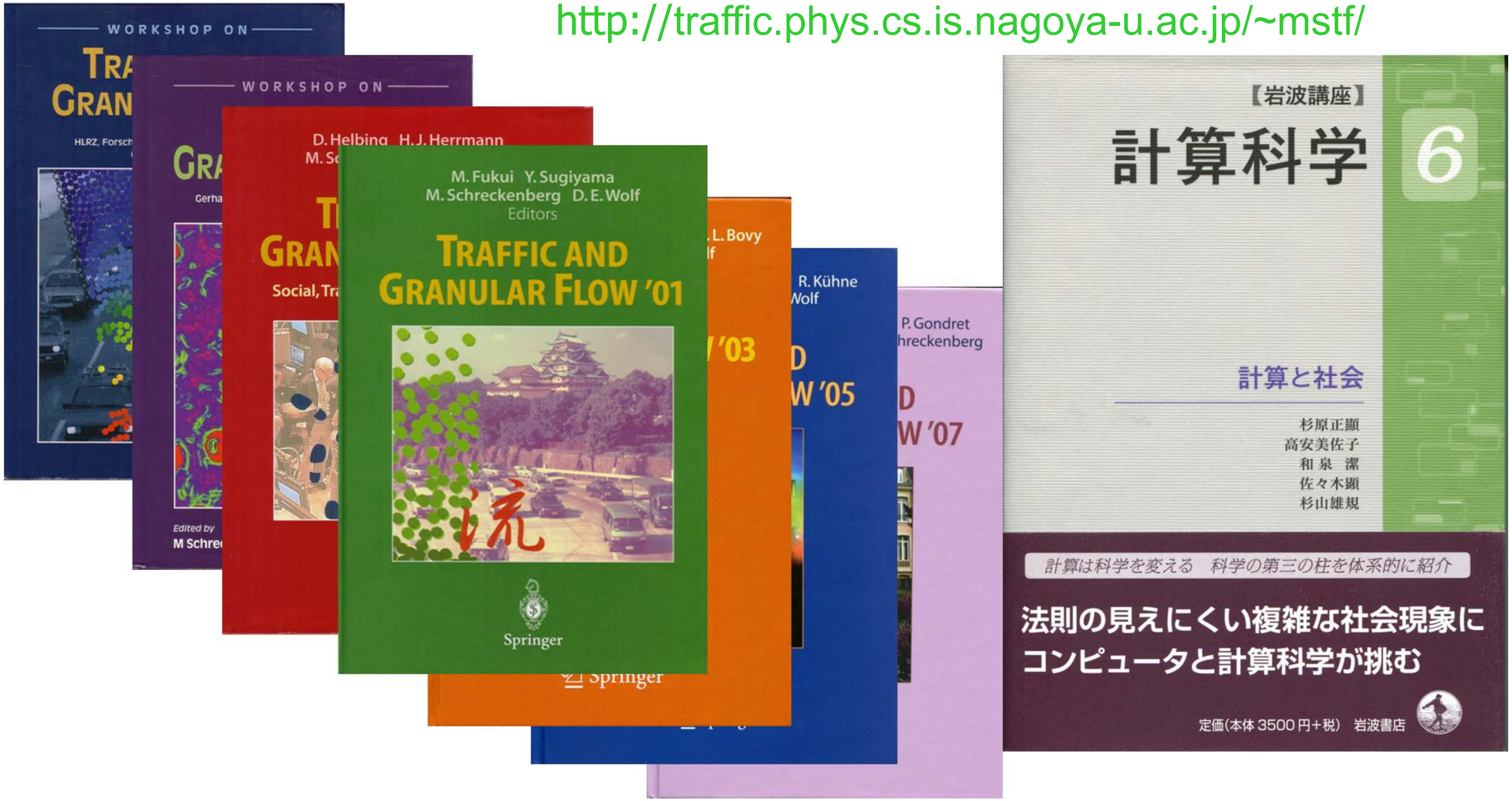
● Physical characteristics

I. Gap from Micro to Macro : **Dynamical phase transition**, Bifurcation
II. Emergence of macroscopic **spatial scale** : Pattern formation
III. Emergence of macroscopic **time scale** : Characteristic time, Rhythm
IV. Fluctuation in macroscopic objects : **Power Law** behaviors

Activity in our Research fields

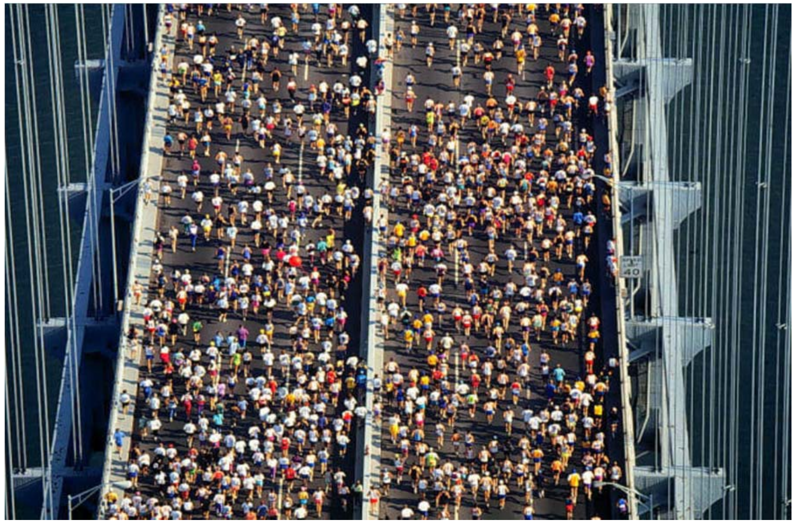
Conferences ・Traffic and Granular Flow '95～(10, every 2 years)
・Pedestrian and Evacuation Dynamics '99～(7, 〃)
・Traffic Flow Symposium in Japan '94～(20, every year)

Access ・Traffic Forum http://www.trafficforum.org
・Mathematical Society of Traffic Flow
http://traffic.phys.cs.is.nagoya-u.ac.jp/~mstf/



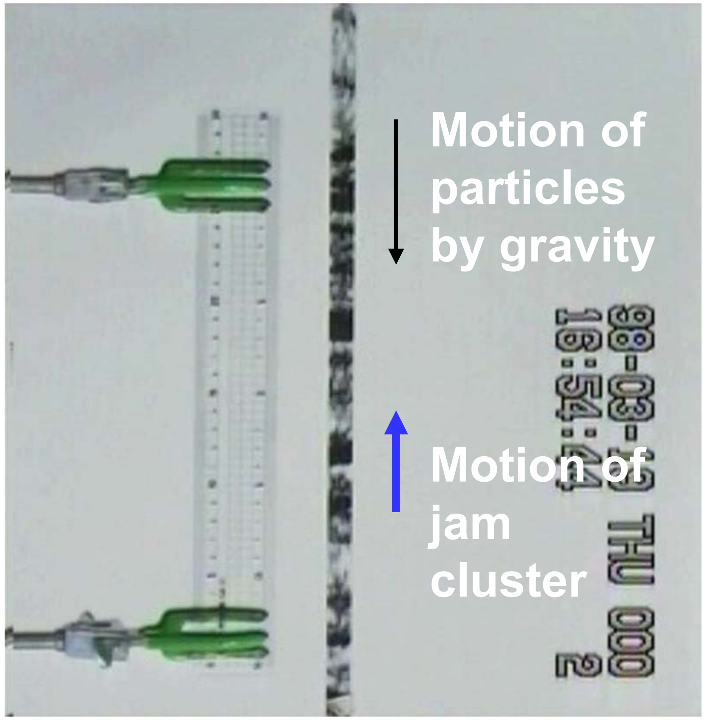
Flow Dynamics of Self-driven Particles

■Traffic flow (high way)



■ Granular flow

(e.g. flow in pipe filled with liquid)



by Nakahara

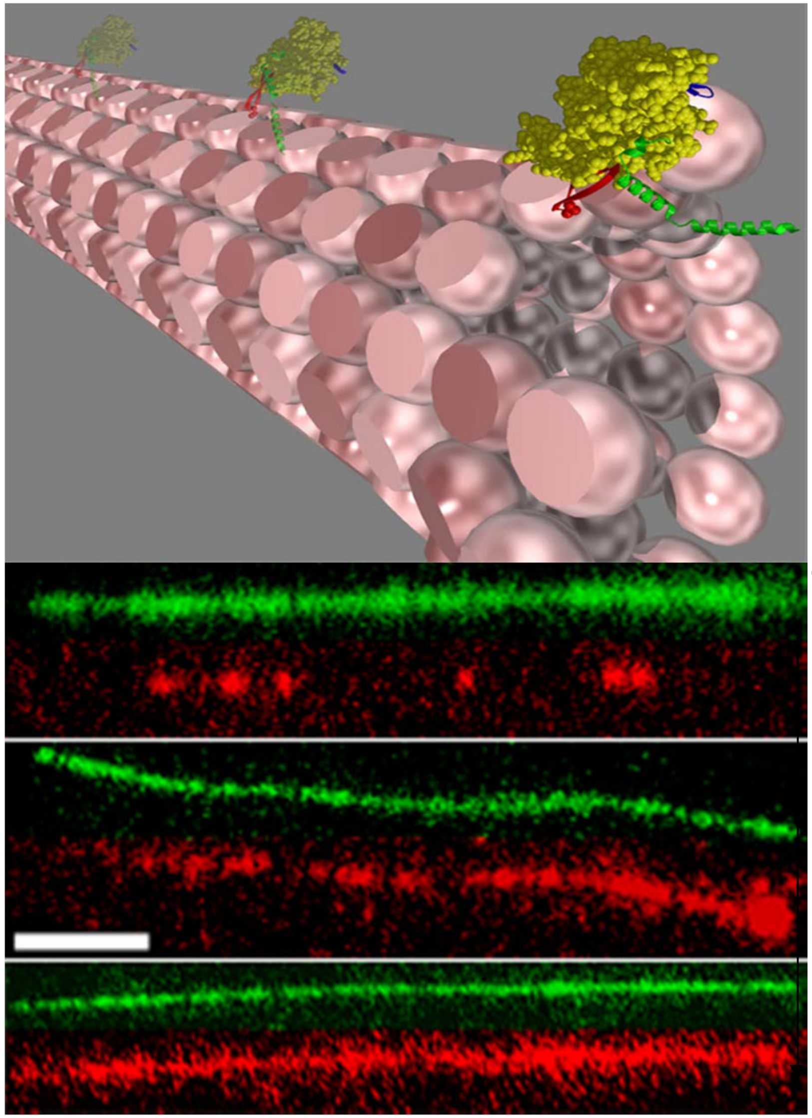
■ Pedestrians and Evacuation Dynamics

■ Ants' trail（chemotaxis）

by H. Nishimori

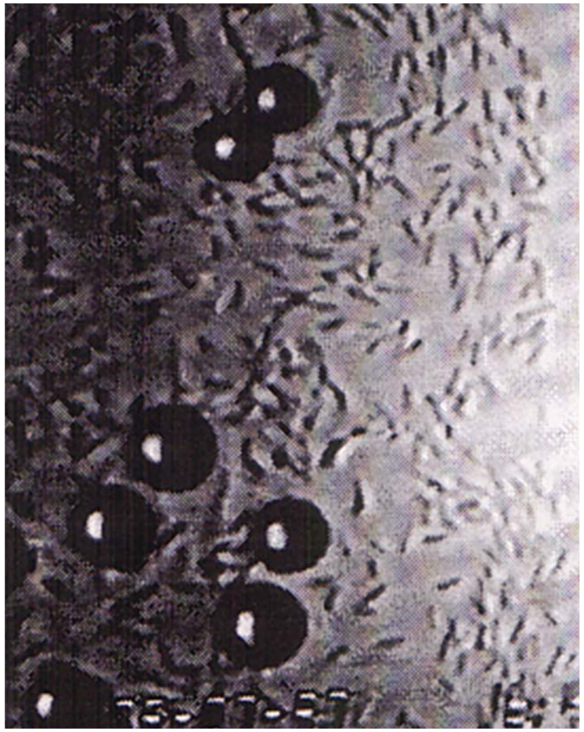
■ Trail formation

■ Jam of molecular mortars

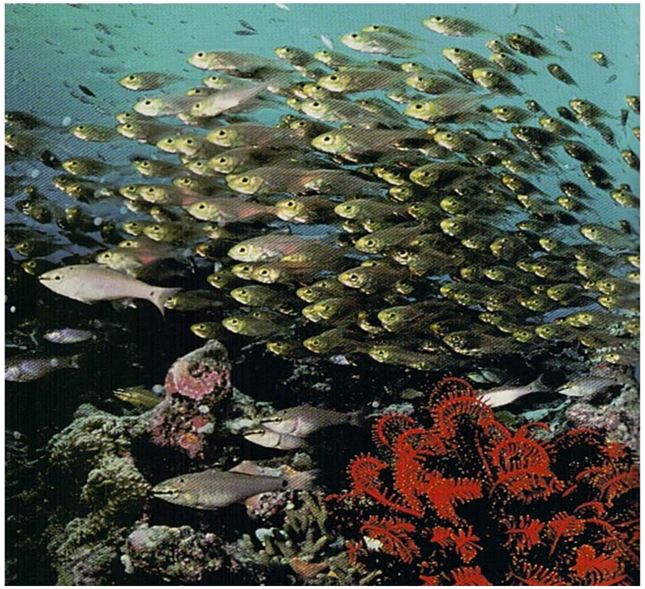


Jam of Kinesin running on microtubule
by Okada, Nishinari

■ Collective bio motions
(e.g. bacteria collony)



■ Group formation of organisms
(e.g. a school of fish)

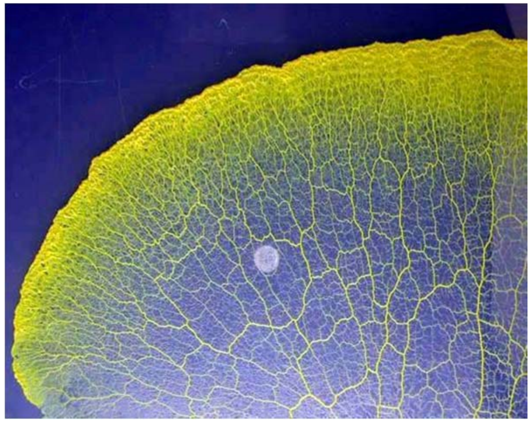


■ Granular flow
(e.g. motion of Barchan Dune)

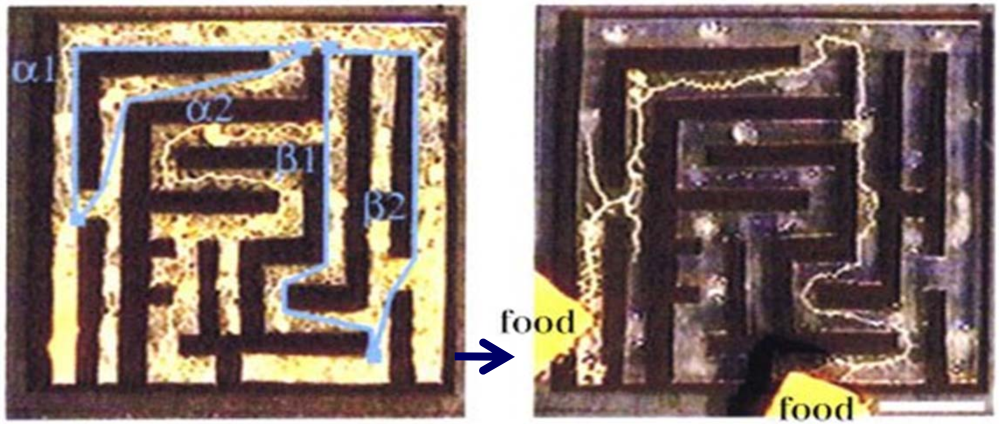
by: H.J. Herrmann

■Group intelligence of Biological system：

(Slime molds can solve the optimal path in a maze: Nature 407(2000) Nakagaki, et.al.)



[1]



[2]

■Control of group motion ⇐ controlled by a tiny stimulus

・Spiral motion of a school of fish

stimulus = (enemy)

・control of sheep

stimulus＝(dog)

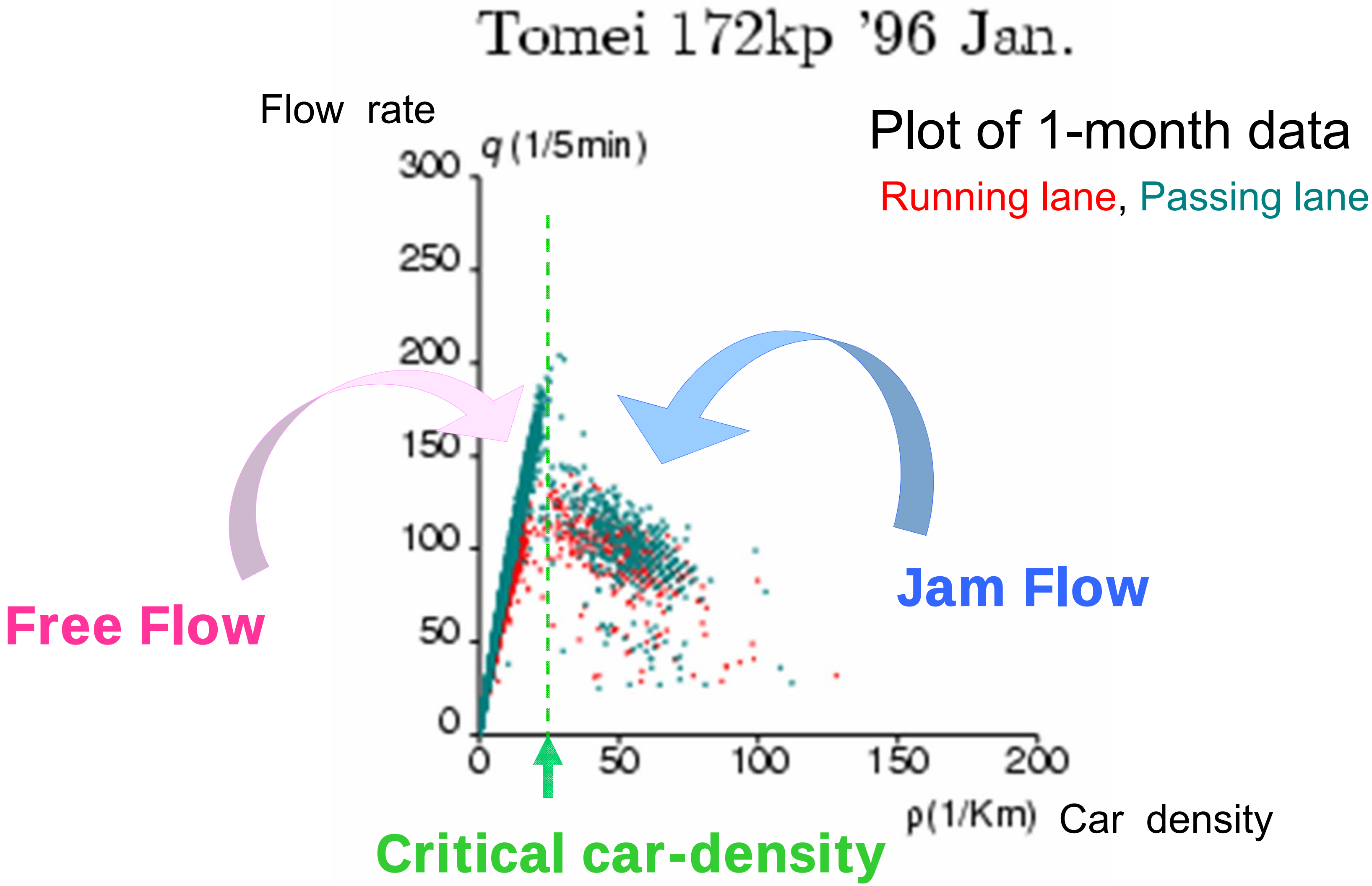
1. Jam formation in traffic flow

the first work of collective motion of self-driven particles

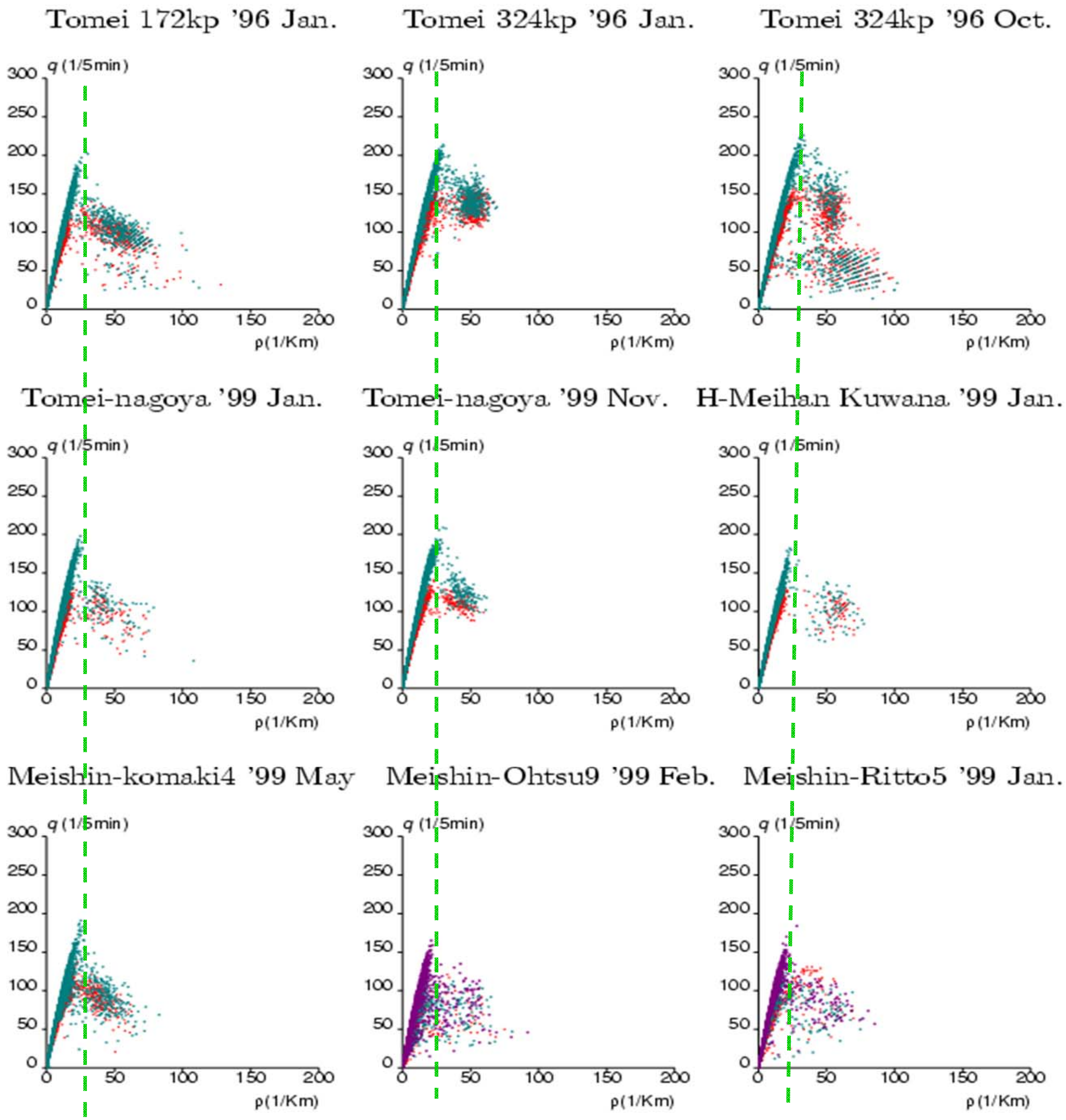
- Fundamental Diagram
- Critical Car-Density
- Velocity of Jam Cluster
- Following behavior
- Flow upstream of Bottleneck

● **Fundamental Diagram** (highway traffic)

Relation (car-density – flow rate) at a fixed measurement point

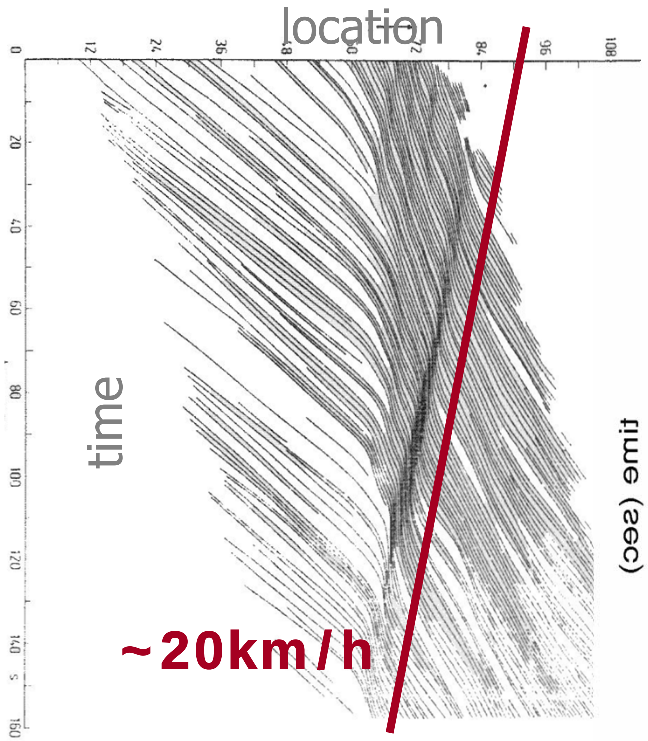


●Flow v.s. car-density relation for several points



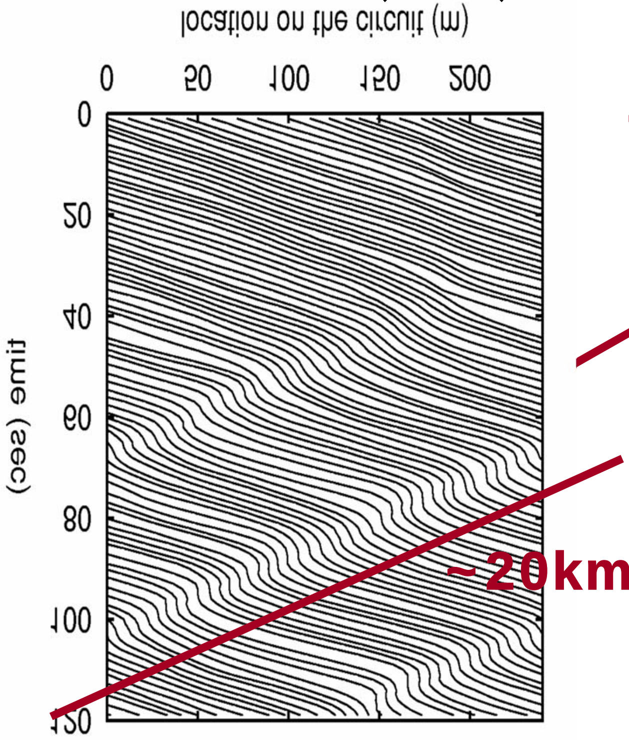
Critical density ~ 25 (1/km)

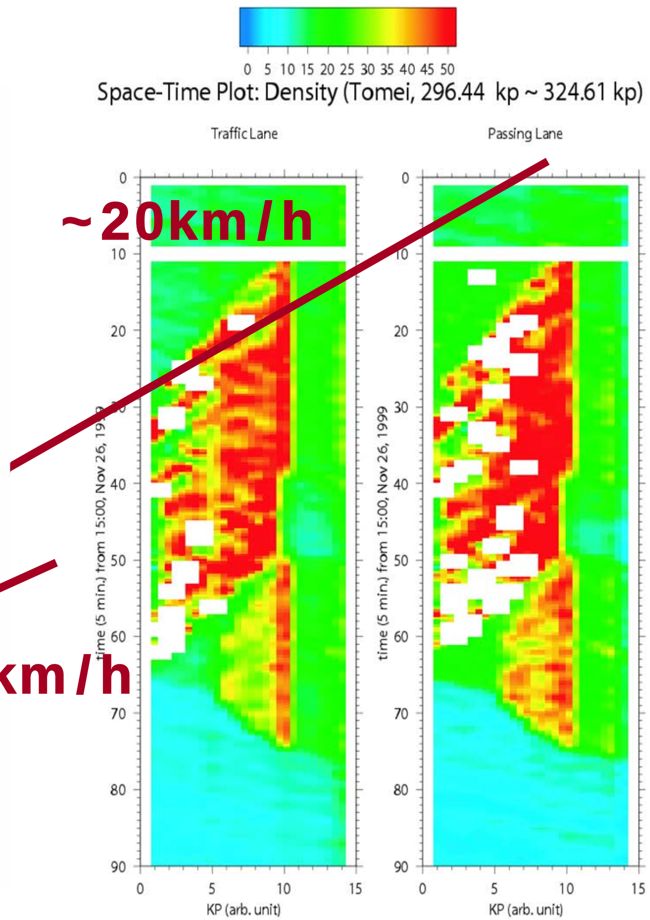
●Velocity of a jam cluster （space-time plot of car motion）



■ Aerial photograph of "phantom jam" : (U.S. A. 1974)

■ Experiment of circular track（2003）





■ Measurement by loop-coil detectors （Jam in upstream of a tunnel）by JTC（1999）

■「Jam cluster（in city highway in Tokyo）moves backward against a traffic flow at the same velocity about **20km/h**」 交通工学通論（by Koshi, 技術書院 1989）

2. Mathematical model for Dissipative System with Asymmetric Interaction (Asymmetric Dissipative System)

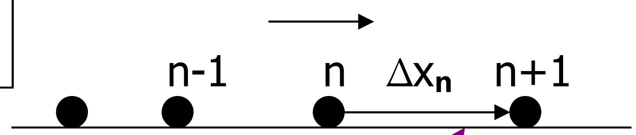
introduced for a model of traffic flow

■ Optimal Velocity Model (1994) (Bando, Hasebe, Nakayama, Shibata, Sugiyama) **Phys. Rev. E 51 (1995) 1035**

Asymmetric Non-linear interacting particles with Dissipative term

$$\frac{d^2 x_n}{dt^2} = a\left\{V(\Delta x_n) - \frac{dx_n}{dt}\right\}$$

the direction of movement



particle : n=1, 2, 3, ･･･

x_n : the position of the nth car

Δx_n : the headway (distance) = $x_{n+1} - x_n$

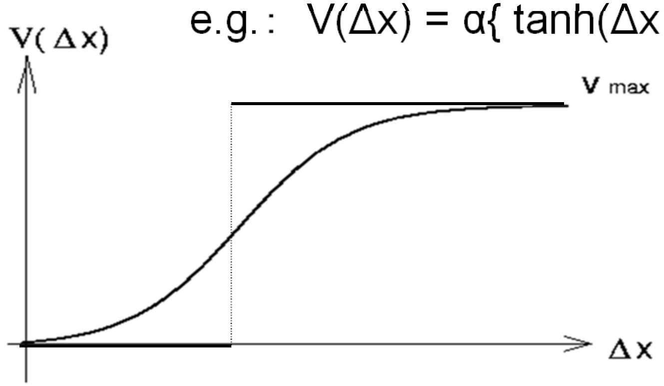
a : sensitivity constant : (1/time or 1/mass), parameter for inertia

$V(\Delta x)$: **OV-function** (→ · the safe velocity for the headway)

non-linear interaction with a **particle in front**:

optimal velocity or asymmetric force

e.g.: V(Δx) = α{ tanh(Δx−d) + tanh d }, Heaviside step function



Interacting particles controlled to optimize each velocity to **V(Δx).**

Asymmetric Non-linear interacting particles with Dissipation

Energy non-conservation

Momentum non-conservation

■ Totally Asymmetric (OVM)

$$\frac{d^2x_n}{dt^2} = a\left\{V(\Delta x_n) - \varepsilon\frac{dx_n}{dt}\right\}$$

(completely asymmetric・nonlinear interaction)

e.g. $V(\Delta x) = \alpha \tanh(\Delta x - d)$

↓ general asymmetric

■ Asymmetric (forward - backward OVM)

$$\frac{d^2x_n}{dt^2} = a\left\{V(\Delta x_n) - W(\Delta x_{n-1}) - \frac{dx_n}{dt}\right\}$$

e.g. $W(\Delta x) = (\alpha \rightarrow \beta)$

Not a Potential Force

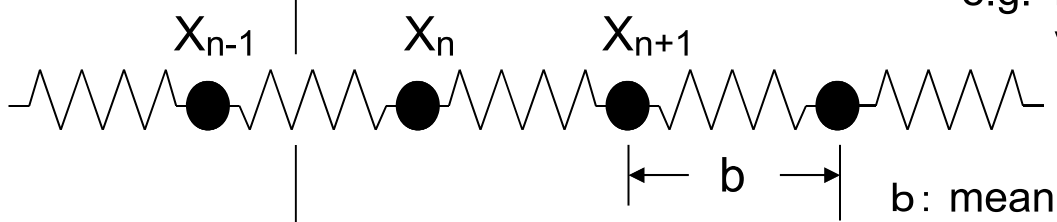
↓ symmetric

■ nonlinear oscillator : (+ viscosity)

$$\frac{d^2x_n}{dt^2} = a\left\{V(\Delta x_n) - V(\Delta x_{n-1}) - \frac{dx_n}{dt}\right\}$$

e.g. Toda lattice

$V(\Delta x) = 1 - e^{-b\Delta x}$



b: mean distance

↓ linear

■ harmonic oscillator: (+ viscosity)

$$\frac{d^2x_n}{dt^2} = a\left\{V'(b)(x_{n+1} - x_n) - V'(b)\cdot(x_n - x_{n-1}) - \frac{dx_n}{dt}\right\}$$

Asymmetric Interaction generates the instability of a homogeneous flow state

■ general OV model

$$\frac{d^2 x_n}{dt^2} = a\left\{V(\Delta x_n) - W(\Delta x_{n-1}) - \frac{dx_n}{dt}\right\}$$

Linear analysis: *y=exp(ikn+zt)*, small deviation, *y beyond a homogeneous flow*

$$z \equiv \sigma - i\omega = ikz_1 + (ik)^2 z_2 + o(k^3)$$, *k~small*

Dispersion relation by long wave-length expansion

■ Totally asymmetric (OV model) :

σ > 0 ⟶ unstable

$$z(k) = ikV'(b) + k^2\left\{\frac{2V'(b)}{a} - 1\right\}V'(b) + o(k^3)$$

b=L/N: average headway (1/density)

W = 0 ↑

negative : stable
positive : unstable

Critical condition

■ **Asymmetric**:

$$z(k) = ik\{V'(b) - W'(b)\} + k^2\left\{\frac{2[V'(b) - W'(b)]^2}{a[V'(b) + W'(b)]} - 1\right\}\frac{\{V'(b) + W'(b)\}}{2} + o(k^3)$$

W = V ↓

■ **symmetric int. + dissipation**:

$$z(k) = -k^2 V'(b) + o(k^3)$$

(absolutely stable)

3. Dynamics of Asymmetric Dissipative System

- Dynamical phase transition

  - many-particles system -

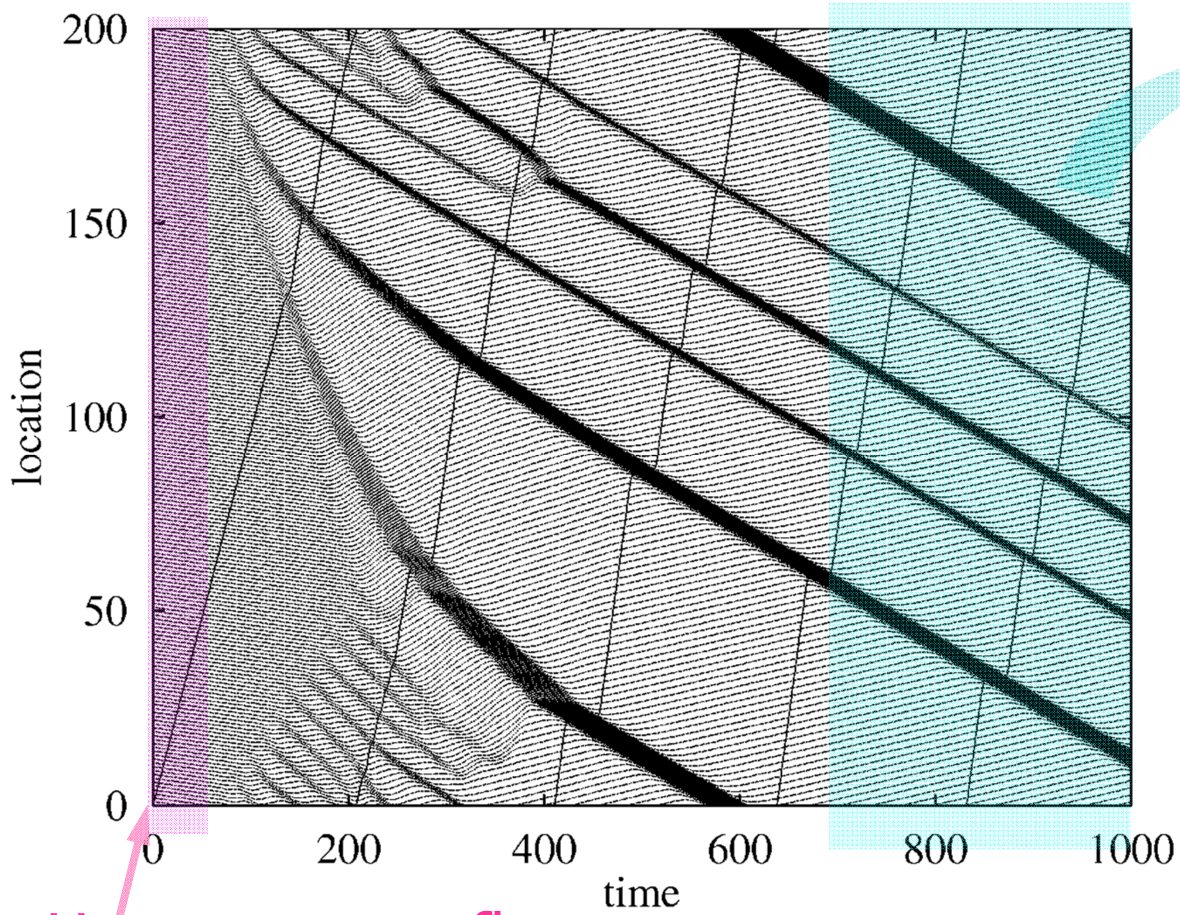
  Bifurcation - stability change of solutions -

- Emergence of moving cluster

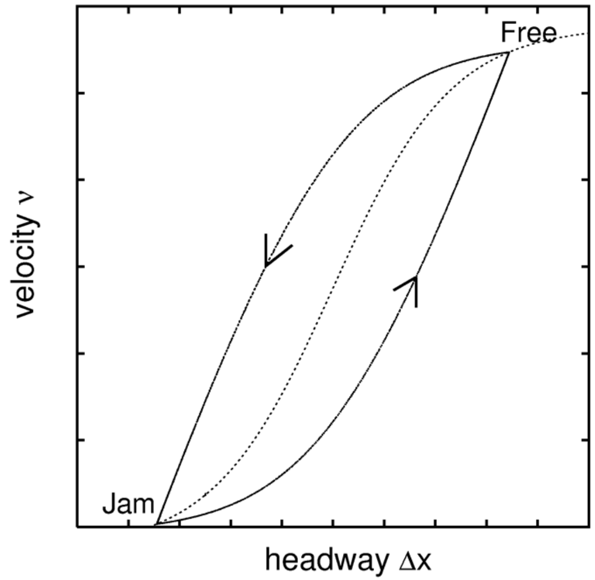
  (macroscopic spatial-scale)

- Induced time (macroscopic temporal-scale)
- Power law behavior

■ Space-time plot for time evolution of forming a cluster



Profile of cluster solution



Homogeneous flow solution : unstable

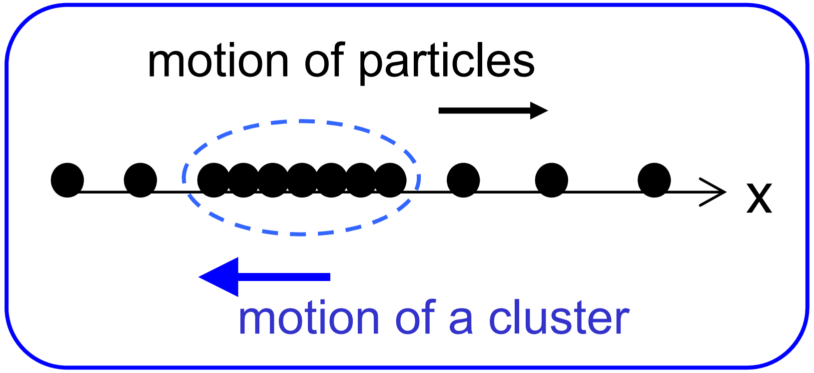
$a < 2V'(b)$

→ Cluster flow solution : stable

■ Dynamical phase transition

Homogeneous flow solution : unstable ➡ Moving cluster flow solution : stable

a < 2V′(b)



■ Emergence of moving cluster

“ Limit Cycle ”

Non-equilibrium balance
of in-and-out flow of particles

⇔ Particles move as the same way with

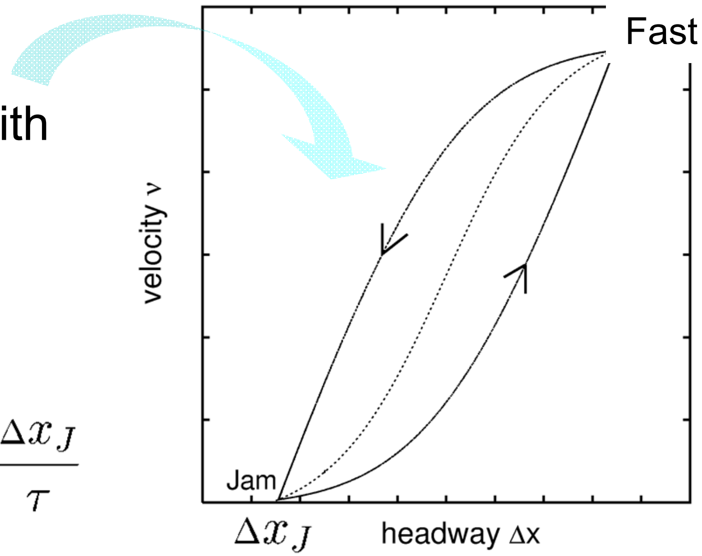
■ Induced time-delay τ

in changing through a cluster.

↓

Forming a macroscopic object
with its own motion : $v_c = -\frac{\Delta x_J}{\tau}$

Profile of cluster flow solution:



4. Experiment, Observational data v.s. Theory of Asymmetric dissipative system

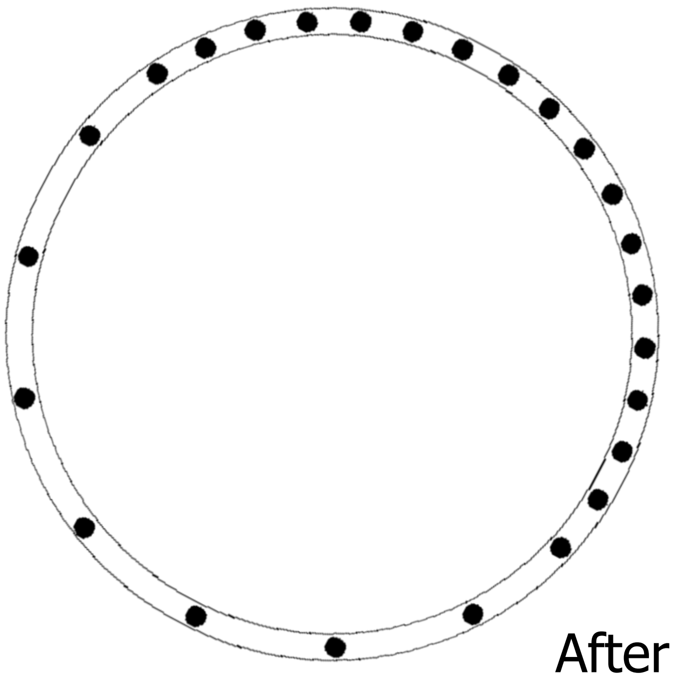
Experiment of Jam formation on Circular track v.s. Simulation of Mathematical model

New Journal of Physics **10** 033001 (2008), Best Paper of the year , *Science*NOW Daily News 28 Mar. 2008, NewScientist, Discovery, YouTube (→shockwave traffic)

Simulation in OV model :

L=230m, N=22. a < 2V'(b)

OV-function : v_{max}=40km/h, Δx>20m

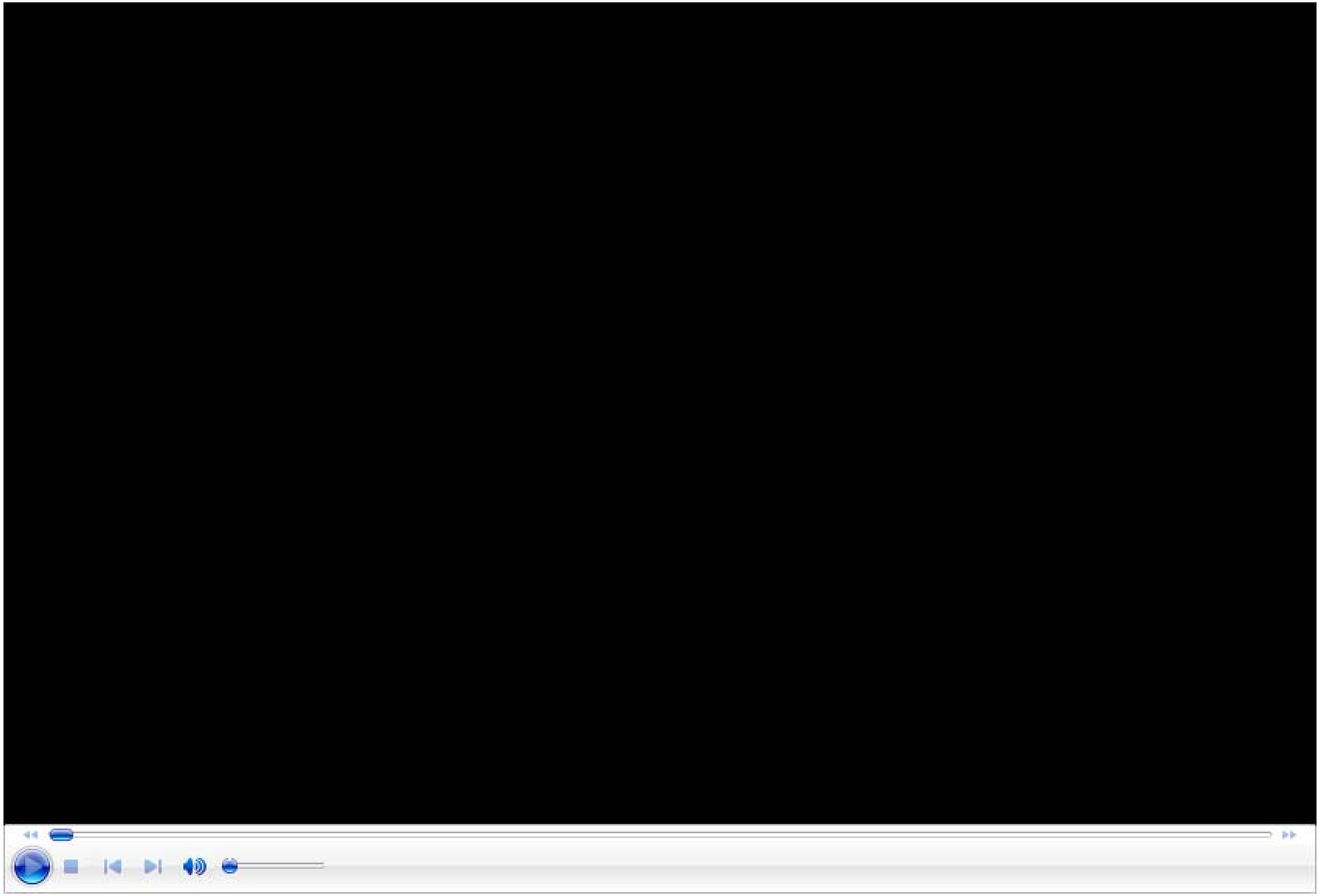


Experiment :

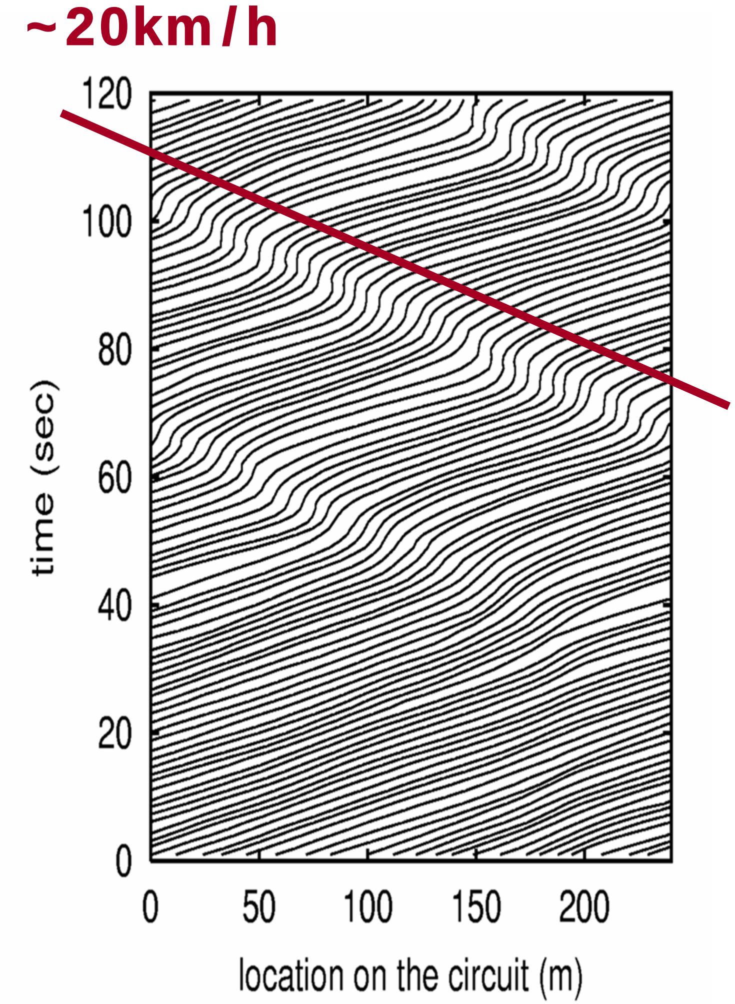
L=230m, N=22.

After 5 min. a Jam cluster was formed and stable.

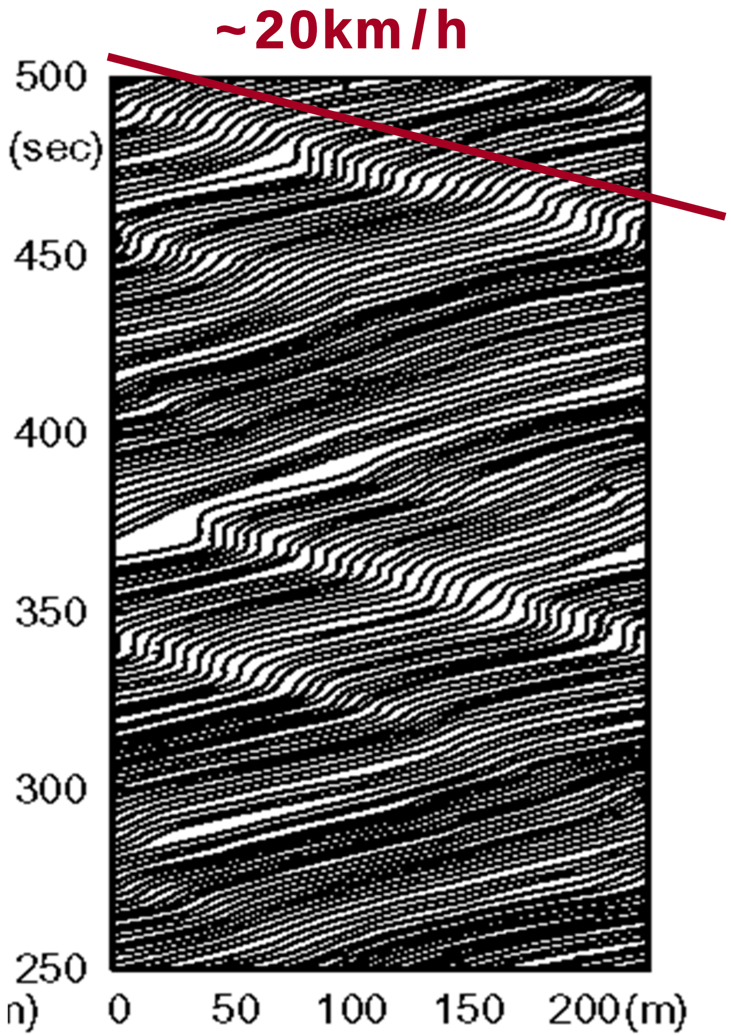
Experiment of Jam formation on Circular track



■ Traces of all cars on the circular track



N= 22, L= 230m



N=23, L=230m

②

Lecture in
Santa Fe Institute

Symmetry Breaking,
Phase Transition
and
Non-Equilibrium
Phenomena

→ santa fe institute lectures
symmetry

RESEARCH HIGHLIGHTS

Selections from the scientific literature

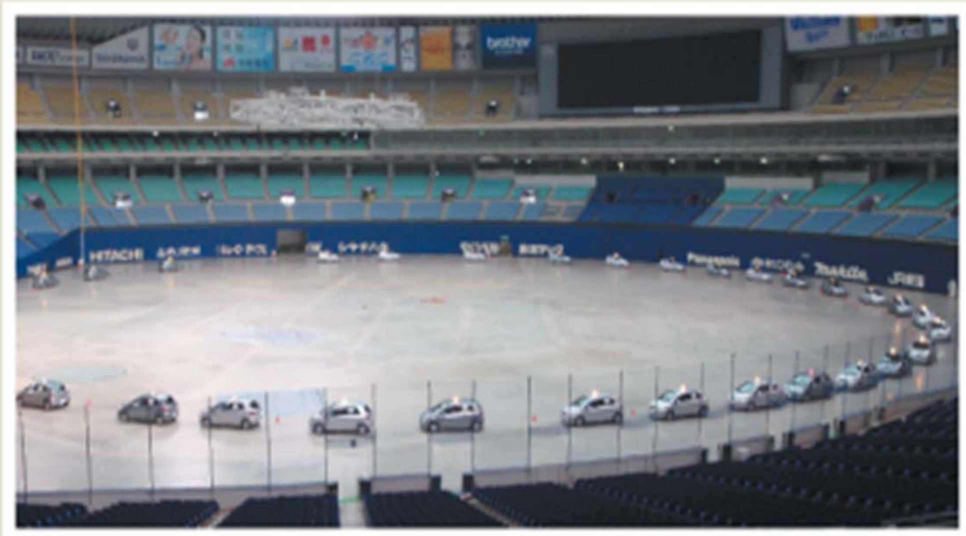
Depression temporarily slowed the planet's warming.

The analysis also suggests that the Montreal Protocol, which phased out chemicals that deplete the ozone layer and trap heat, has helped to slow warming in recent decades.

Nature Geosci. http://doi.org/p2b (2013)

For a longer story on this research,

①



PHYSICS

Traffic jams follow the laws of physics

Traffic congestion closely resembles the physics of phase transitions, such as when ice melts or a metal becomes superconducting.

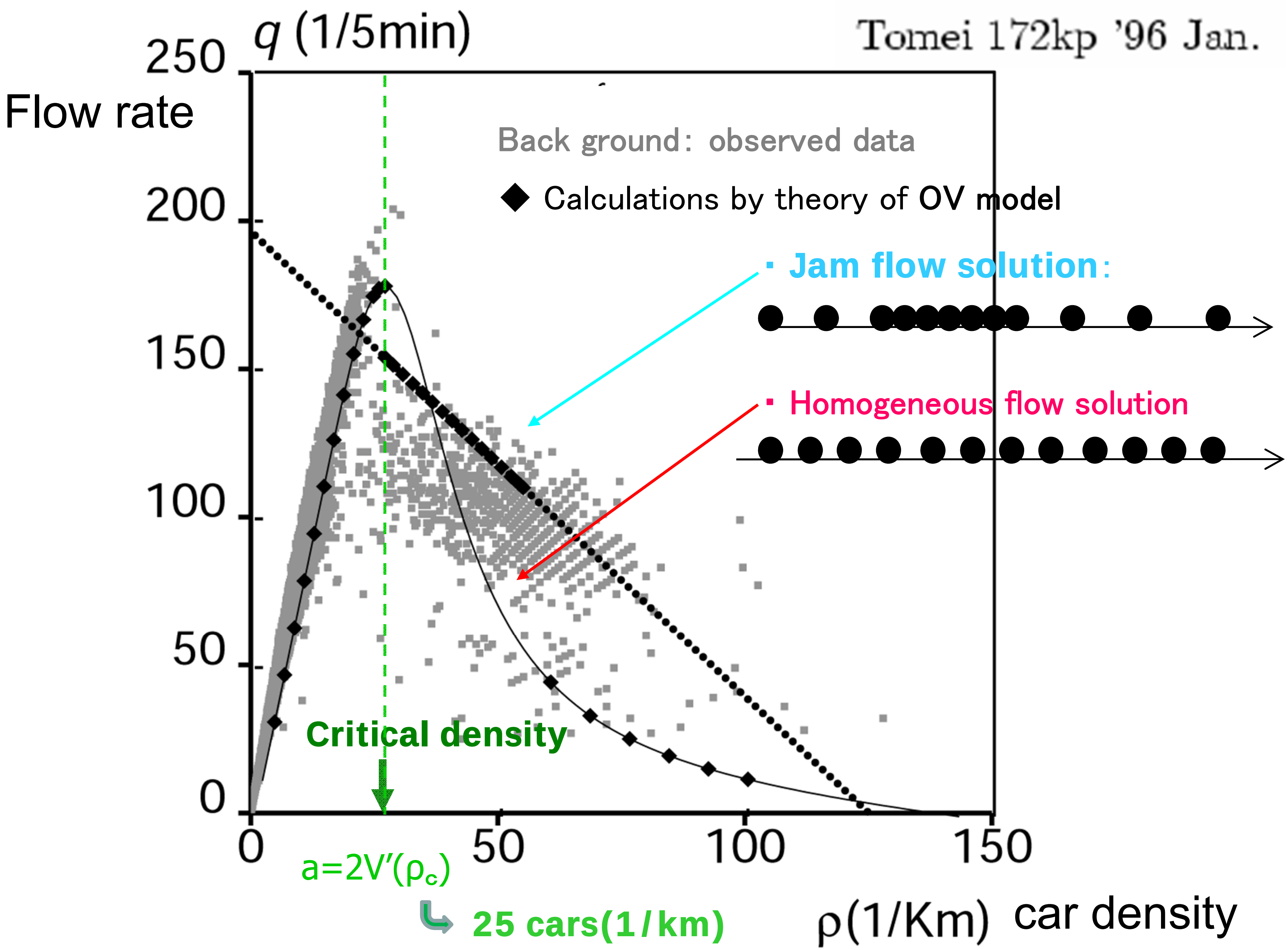
Shin-ichi Tadaki at Saga University in Japan and his colleagues used a high-resolution laser scanner to track cars travelling around an empty indoor baseball stadium, then analysed those data as if they were studying phase transitions in a material. They found that above a critical density of cars, traffic flow became unstable and changed from free-flowing to a jam.

Scaled up, that density value fits with those seen on real-world motorways, the authors say.

New J. Phys. 15, 103034 (2013)

New J. Phys. 15, 103034 (2013)

Observational data in highway v.s. OV Theory



5. Mathematical aspects of Asymmetric Dissipative System

- Dynamical phase transition

  - many-particles system -

  Hopf Bifurcation

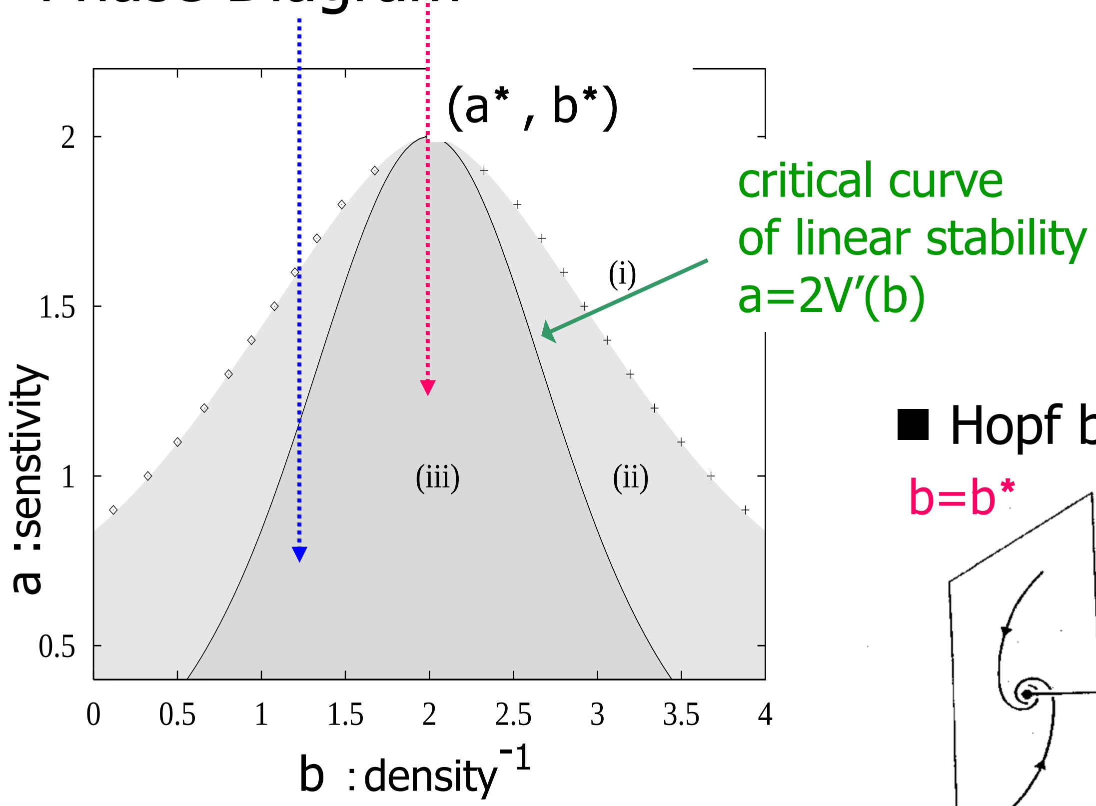
  - stability change of solutions -

- Emergence of moving cluster

  (macroscopic spatial-scale)

Property of Dynamical phase Transition

■ Phase Diagram

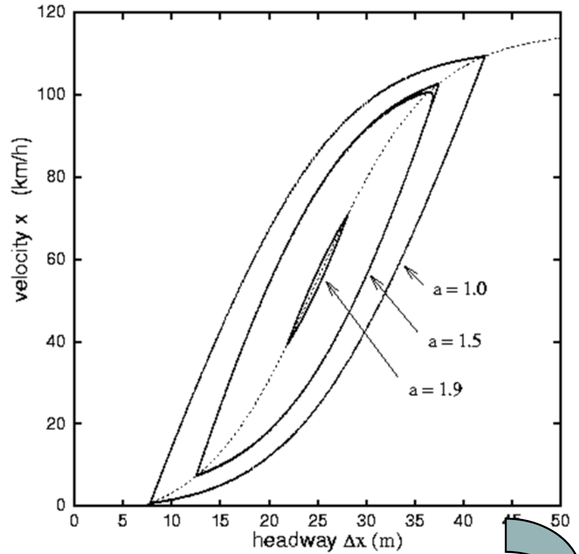


i) homogeneous flow : stable

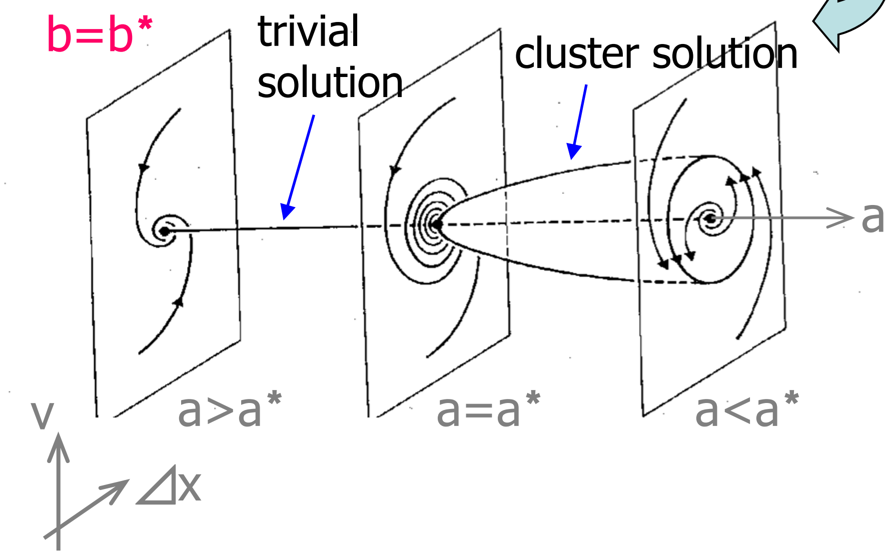
ii) homogeneous flow, jam flow :
both stable (Bistable phase)

iii) jam flow : stable

a **- dependence of "limit cycle"**



■ Hopf bifurcation



b≠b* : subcritical Hopf bifurcation

■ Asymmetric Interaction $\xrightarrow{\text{Yes !}}$ Hopf-bifurcation

Physical. Review. E 80, 026203 (2009)

$$\frac{d^2x_n}{dt^2} = a\left\{V(\Delta x_n) - W(\Delta x_{n-1}) - \frac{dx_n}{dt}\right\}$$

The linear equation of motion for small deviation, y beyond a homogeneous flow.

$y_n = exp(ink+zt)$, $\quad z \equiv \sigma - i\omega$

$$\begin{cases} \sigma^2 - \omega^2 = a(V'(b) + W'(b))\cos\theta - a\sigma \\ -2\sigma\omega = a\left(V'(b) - W'(b)\right)\sin\theta + a\omega \end{cases}$$

V = *W* → *ω = 0* only, *at bifurcation point: σ = 0*

i.e. No bifurcation

V≠ W → *Eq. of* ω^2 **,** which always has positive solution in $a < 2V'(b)$.

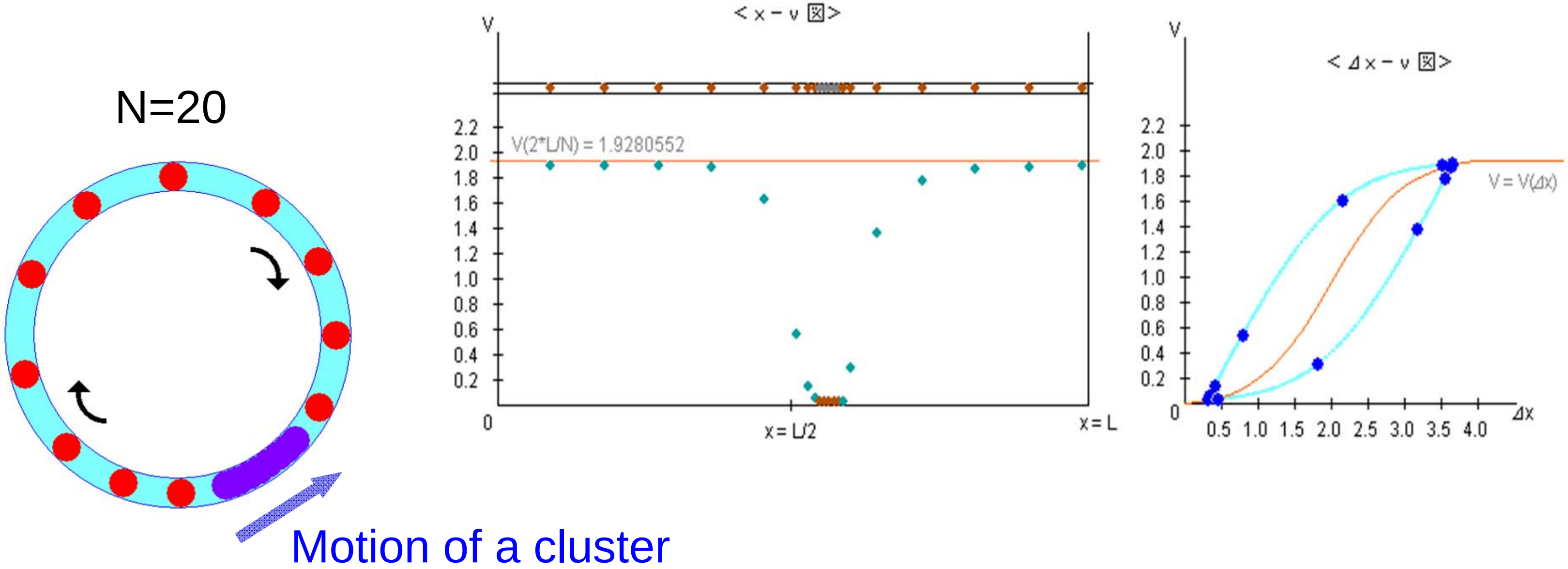
⇒ *±ω are conjugate solutions at σ = 0*

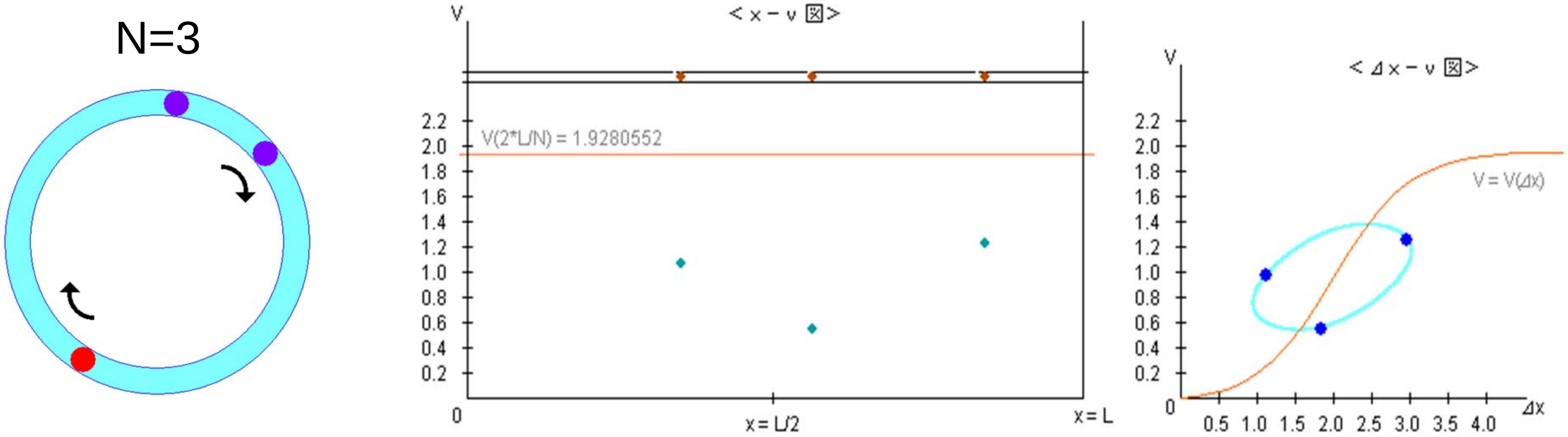
i.e. Hopf-bifurcation

6. Specific Properties in Asymmetric Dissipative System

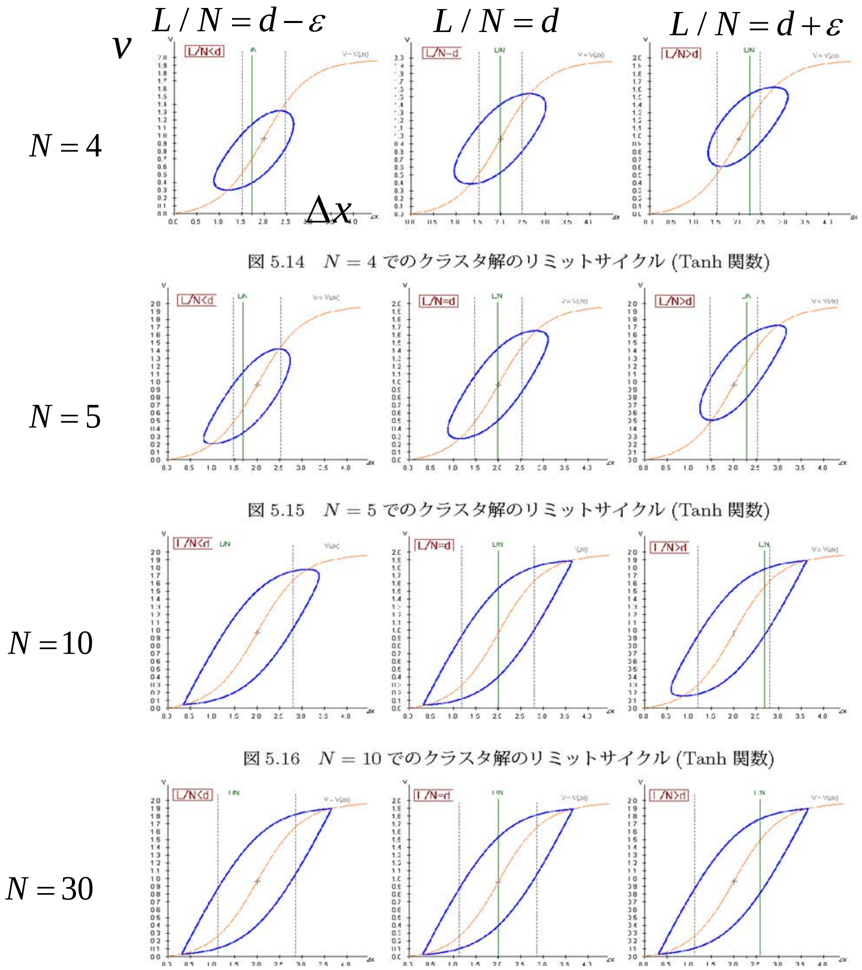
- N (# of Particles) - dependence
- Properties of Emerged Objects

■ Cluster emerges even in N=3 .

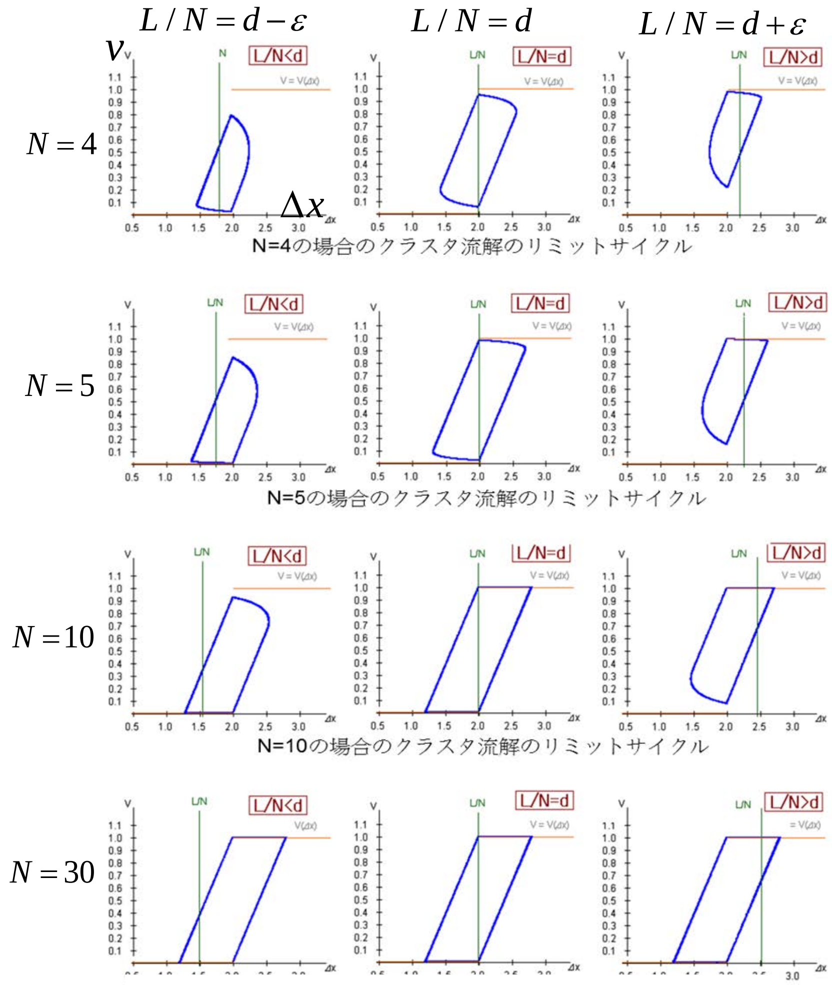




Limit cycle(Jam flow) for **N(#of particles)** and **L/N(density)** , **a=1**



e.g. V(Δx) = α{ tanh(Δx－d) ＋ tanh d }



e.g. V(Δx) = Heaviside Step function

d : the inflection point (1/density)

Asymmetric
Dissipative *System* ⟶ $e^{-Na\tau}$

■ Limit cycle solutions **converges very rapidly** to **the universal solution**, independent on N and L/N.

Universality of Jam

7. Instability in 2-dim. System and Group formation

- 2-dimensional OV model
- Higher dimensional modes
  - longitudinal mode
  - transverse mode

  (the same as 1-dim)

  gas・liquid・solid
  (continuum system)
  - elliptically polarized mode

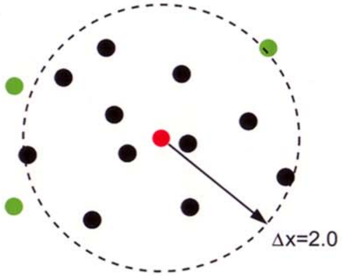
Granular, discrete particles

2-dimensional OV Model for Biological Motion or Pedestrians

Basic idea: An organism maintains an optimal velocity which depends on distances to others

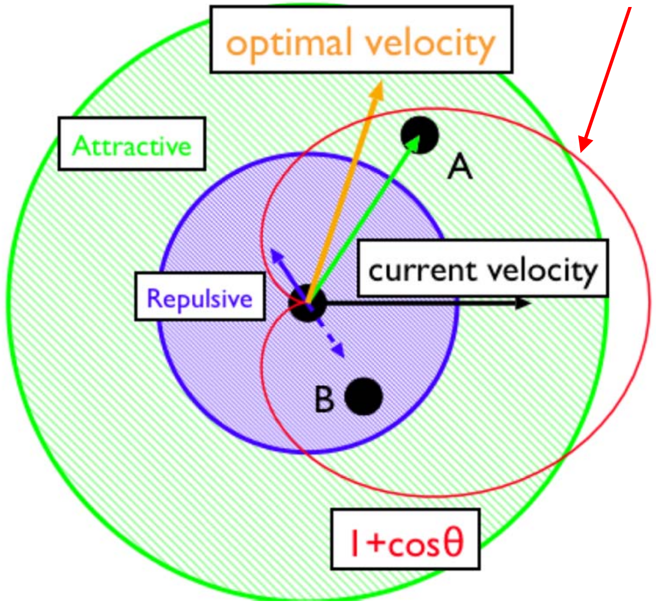
Eq. of motion:

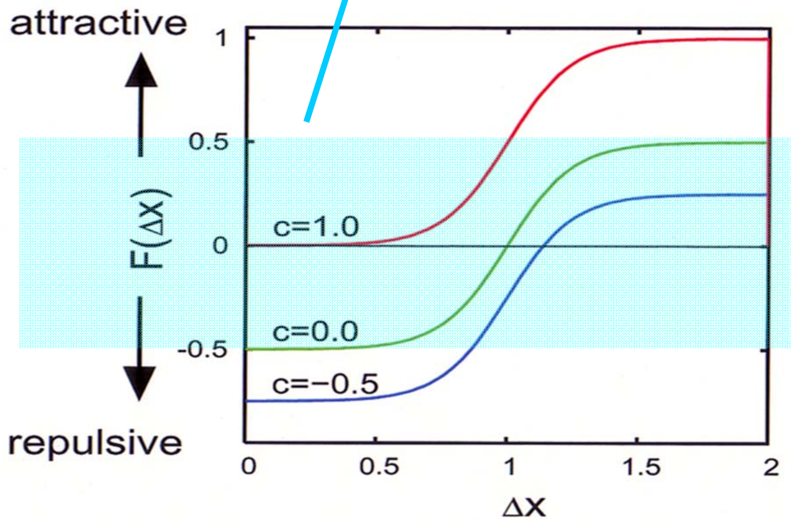
$$\frac{d^2}{dt^2}\mathbb{x}_n(t) = a\left[\sum_k \mathbb{F}(\Delta\mathbb{x}_{kn}(t)) - \frac{d}{dt}\mathbb{x}_n(t)\right]$$



$$\mathbb{F}(\Delta\mathbb{x}_{kn}) = \begin{cases} \mathbb{n}_{kn}\left(\frac{1+\cos\theta}{2}\right)\left(\frac{\tanh 4(\Delta x_{kn}-1)+c}{2}\right) & : \Delta x_{kn} < 2 \\ 0 & : \Delta x_{kn} \geq 2 \end{cases}$$

Asymmetry

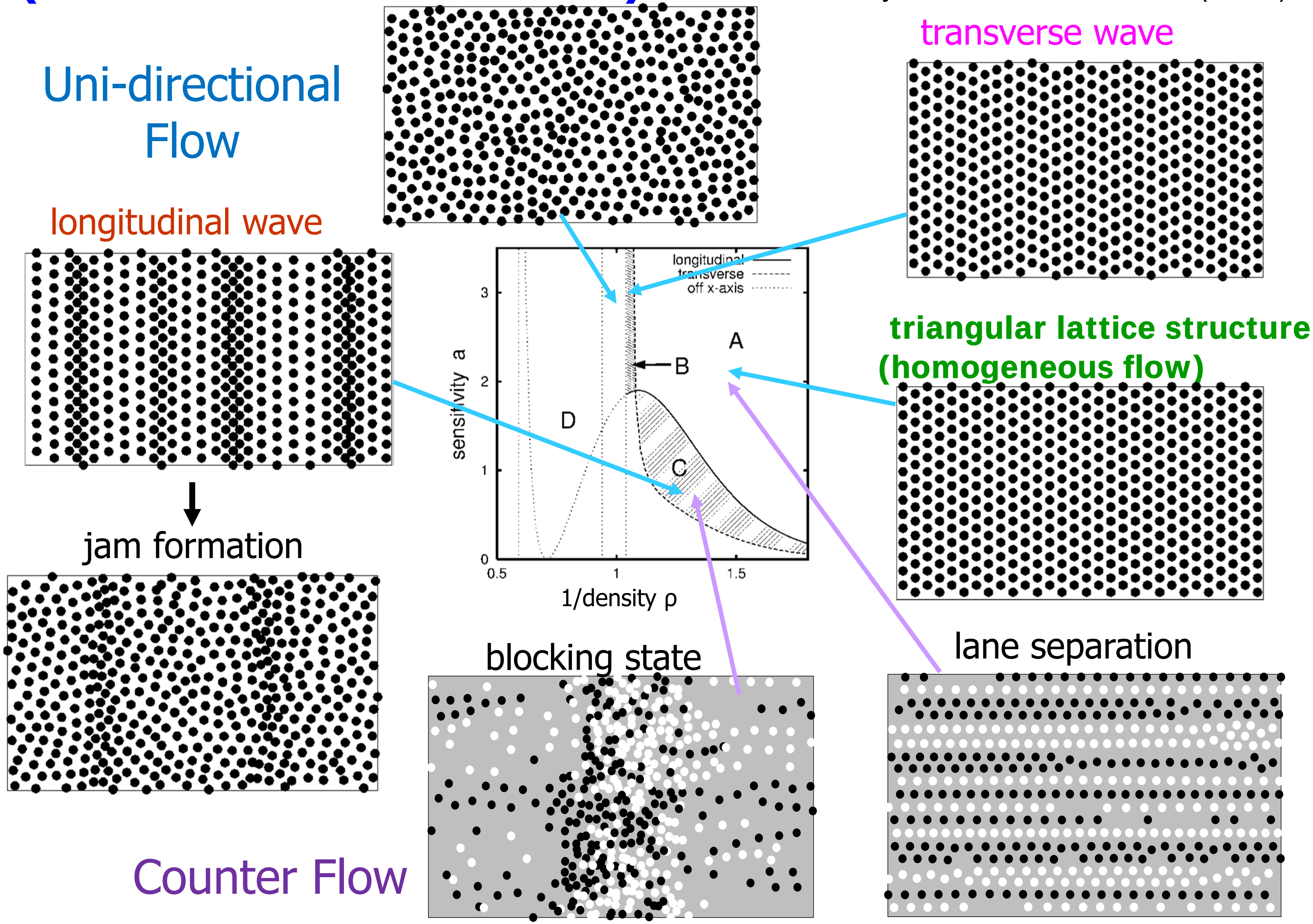




Phase diagram and Behaviors of the exclusive model

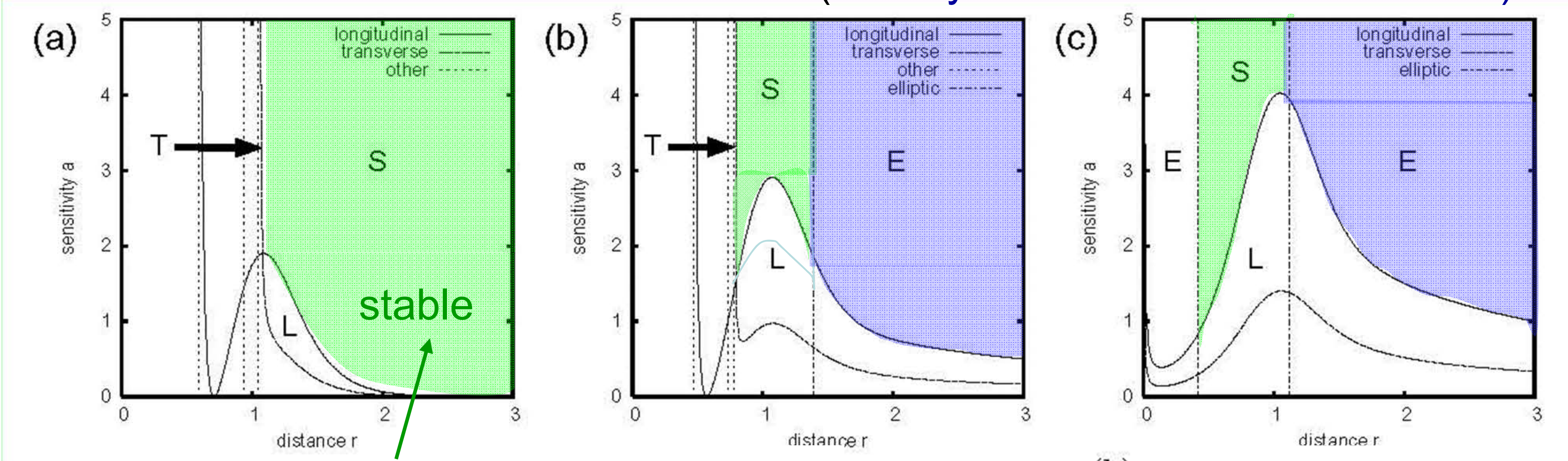
(Pedestrian flow in a corridor) C=－1

Phys. Rev. E **71**, 036121 (2005)

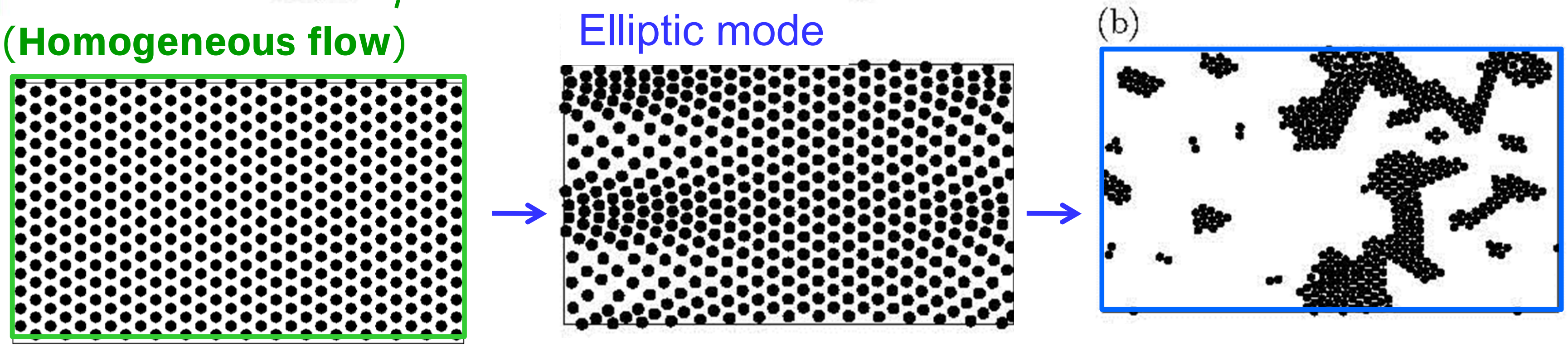


Group formation is induced by Elliptically polarized mode in C ≠ -1

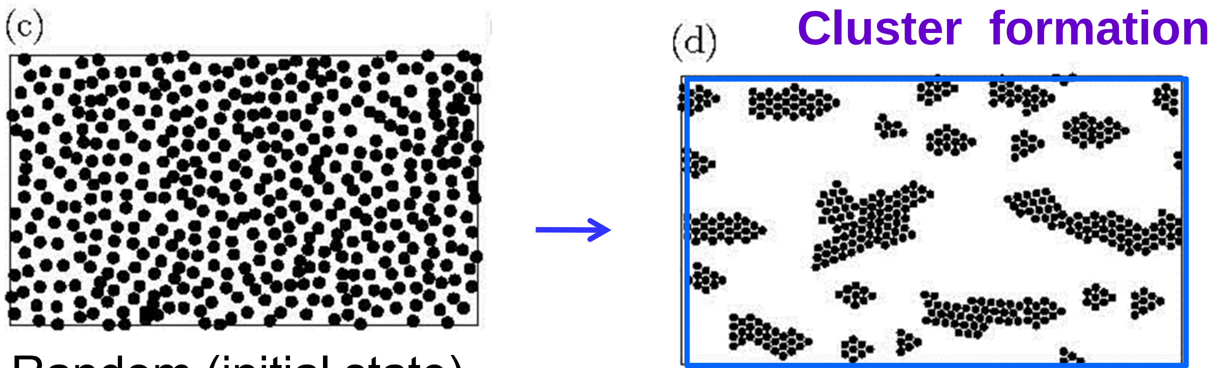
c= -1 (if a tiny attractive interaction exists!)



(Homogeneous flow) Elliptic mode



Cluster formation



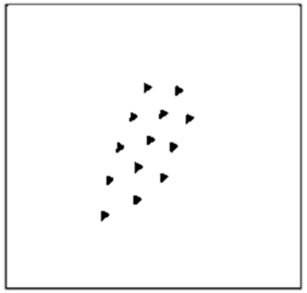
Random (initial state)

Phys. Rev. E **77**, 016105 (2008)

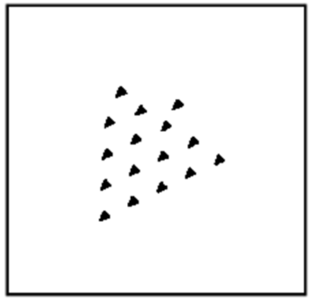
Random motion and Variation of Emerged Object of Deterministic Motions

2-dimensional OV model

(b) $c = 0.0,\ a = 1.0$



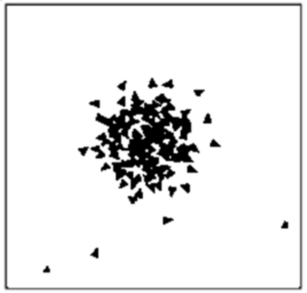
(c) $c = -0.5,\ a = 3.0$



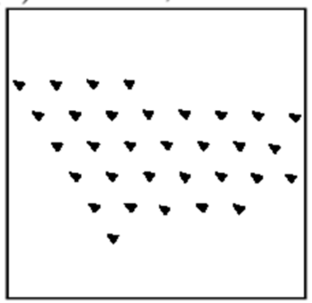
(a) $c = 0.0,\ a = 1.0$

(b) $c = 1.0,\ a = 3.0$

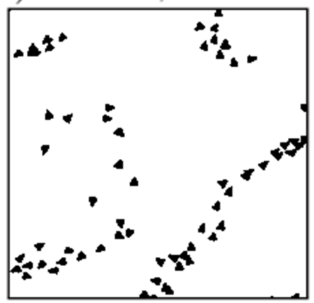
(e) $c = 1.0,\ a = 3.0$



(f) $c = -0.5,\ a = 3.0$



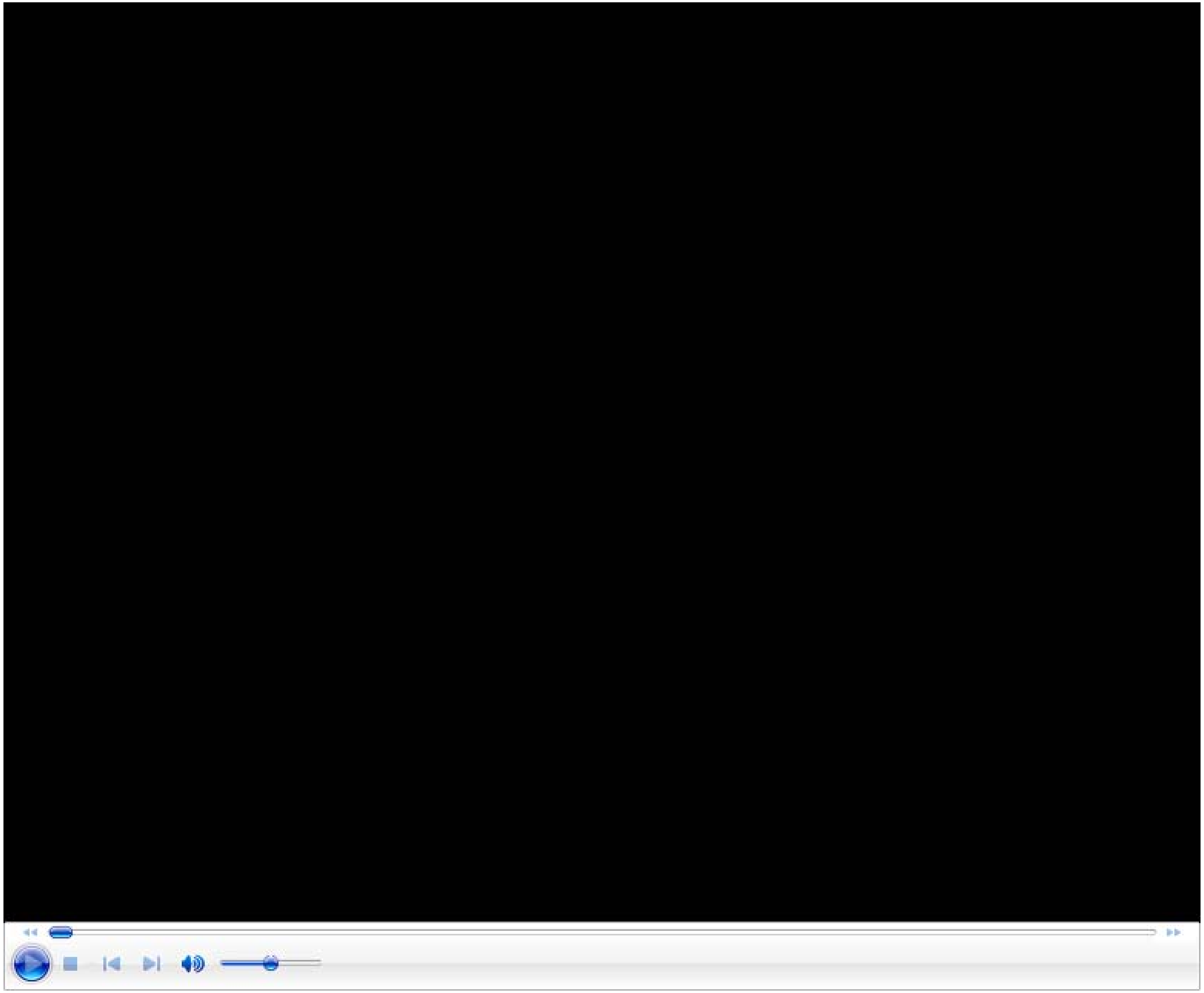
(d) $c = -0.5,\ a = 3.0$



(e) $c = 1.0,\ a = 3.0$

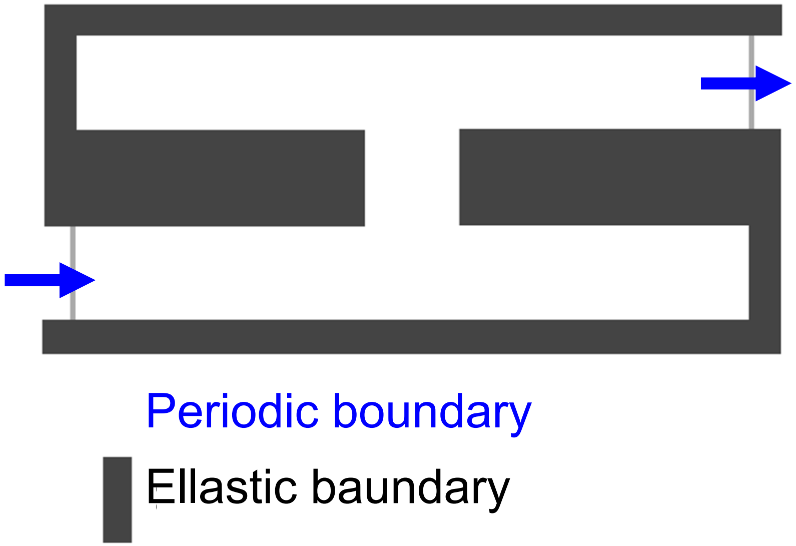
Rapid Response to stimulus

2d-OV Simulations

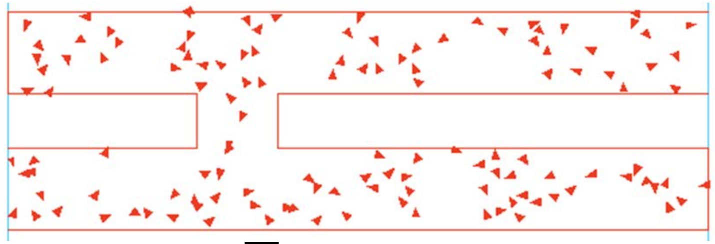


Formation of solutions for a simple maze

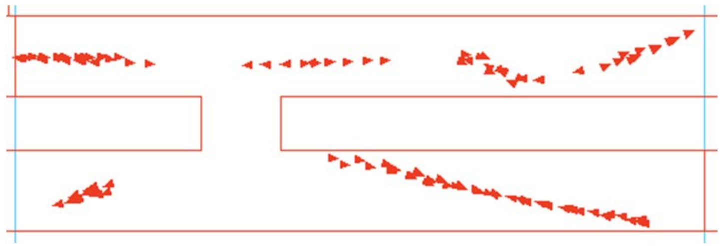
(a=20.0,N=128)



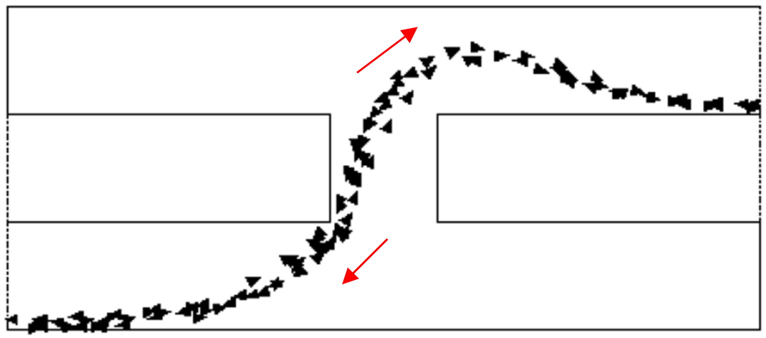
Inital state (randam motions)



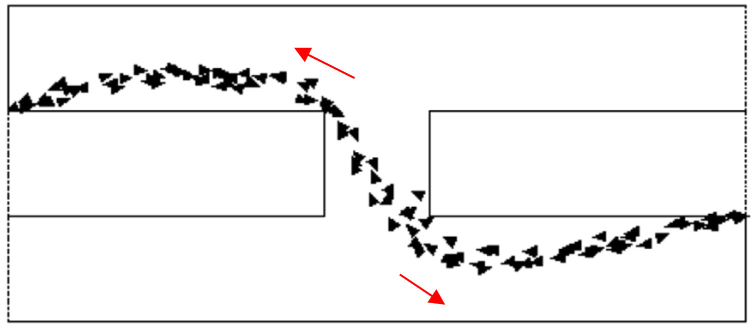
Intermediate state (creating strings)



Solutions of optimal path



i) Soluiton of periodic baundary



ii) Solution of ellastic boundary

8. Coarse Analysis

Analysis of macroscopic dynamical state in collective motions of microscopic variables

- Diffusion Map
- Kantorovich metric space
- Equation-free method

e.g.) Motions of 2-d OV

(Search for the solution of a maze)

in Kantorovich space

→ see the poster No.9

by R. Ishiwata (Nagoya Univ.)

9. Summary

Non-equilibrium Dissipative System with Asymmetric Interaction	Energy-momentum Conserved System with Symmetric Interaction
■ Inseparable connection between Asymmetry of Interaction and Dissipation → Dynamical behavior ■ Density is a control parameter.	■ Dissipation → Static state. ■ Density can not be a control parameter.
N(# of particles)-dependency	
■ Instability and Phase transition emerges in small N. (N≧3 in OVM) ■ Small- N is large enough number as many-body system.	■ Phase transition appears in large N. (strictly, in N→∞)
Emerged macroscopic Objects	
■ Similarity between Temporal and Spatial structures ■ Rapid Response to stimulus ■ High Degeneracy in “local symmetry”	■ Slow Response ■ Degeneracy in global symmetry

ナゴヤドーム渋滞形成実験

渋滞はなぜ発生するのか -その数理と実証実験映像-

- Y. Sugiyama（Nagoya University）
- M. Fukui（Nakanihon Automotive College）
- T. Yoshida（〃 ）
- M. Kikuchi（Osaka University）
- S.-i. Tadaki（Saga University）
- A. Nakayama（Meijyo University）
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