Chases and Escapes with Groups

Toru Ohira
Graduate School of Mathematics
Nagoya University, Japan

This research has been supported by
The Kayamori Foundation of Informational Science Advancement
#K25-XVII 422

Group Chase and Escape (Kamimura and Ohira, 2010)

(A)

* Old problem from 17th Century: Tractrix
* Applications to real world problem
* But, mostly, one-to-one cases analyzed

(B)

* More functioning for the "particles":
  Granular Physics -> Traffic Problems -> Group Chase and Escape
* Swarms of insects, animals, fish, etc.

Our Approach:

Consider the problem of chase and escape in groups from complex physics perspectives by fusing approaches from (A) and (B).

Example of Chase and Escape

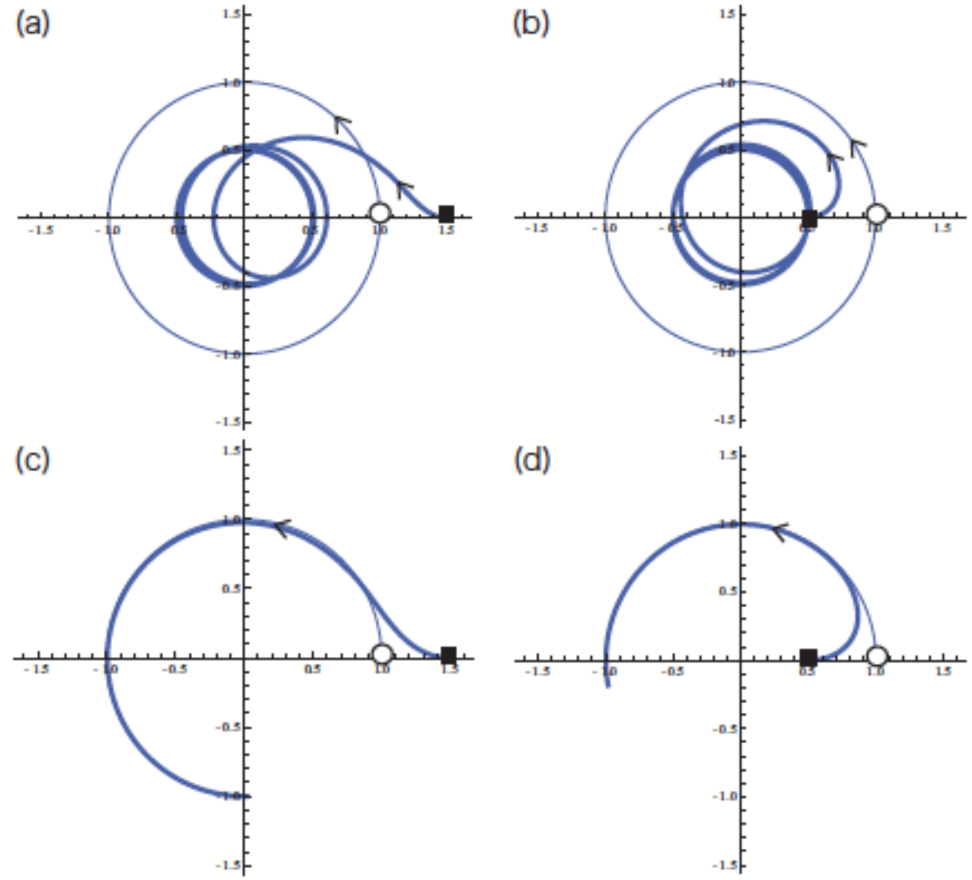


図 1　等速度で円運動をする逃避者（細線）と追跡者（太線）の軌跡. それぞれの始点は白丸と黒四角で表されていて, 追跡者の速度ベクトルは常に逃避者の位置を向いているという問題設定になっています. 追跡者と逃避者の速さの比は $(a,b)1:2$（追いつけない）, $(c,d)1:0.95$（追いつく）としてあります.　なお, (a,b) において追跡者の軌跡が逃避者と同様に円となり, その半径が速度比と同じ $1:2$ となるのは証明されています.

Model

Field: **Square Grid with a periodic boundary condition (100 x 100).**
Chasers: **N_C (trying to move to the closest target).**
Targets: **N_T (trying to move away from the closest chaser).**

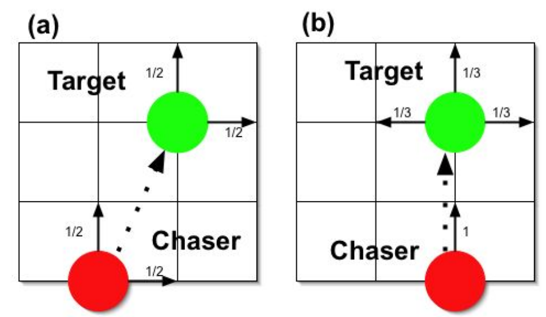


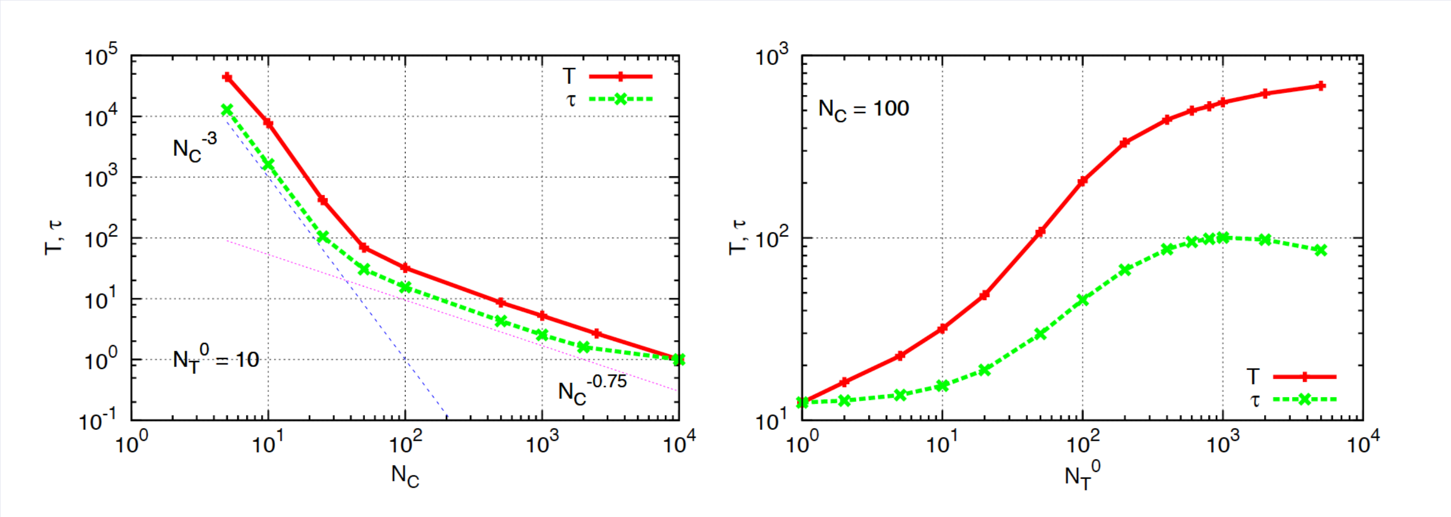
Figure 1. Hopping rules for chasers and targets. While chasers hop to close in on their nearest targets, targets hop to evade their nearest chasers. Dotted arrows from chaser to target indicate that chaser hops to close in on target. Solid arrows show possible hopping directions with indicated probabilities. (a) Generally, they have two choices. (b) When chasers or targets are in same x or y-axis, chasers have one choice, while targets have three choices.

Time to Catch All Targets: T

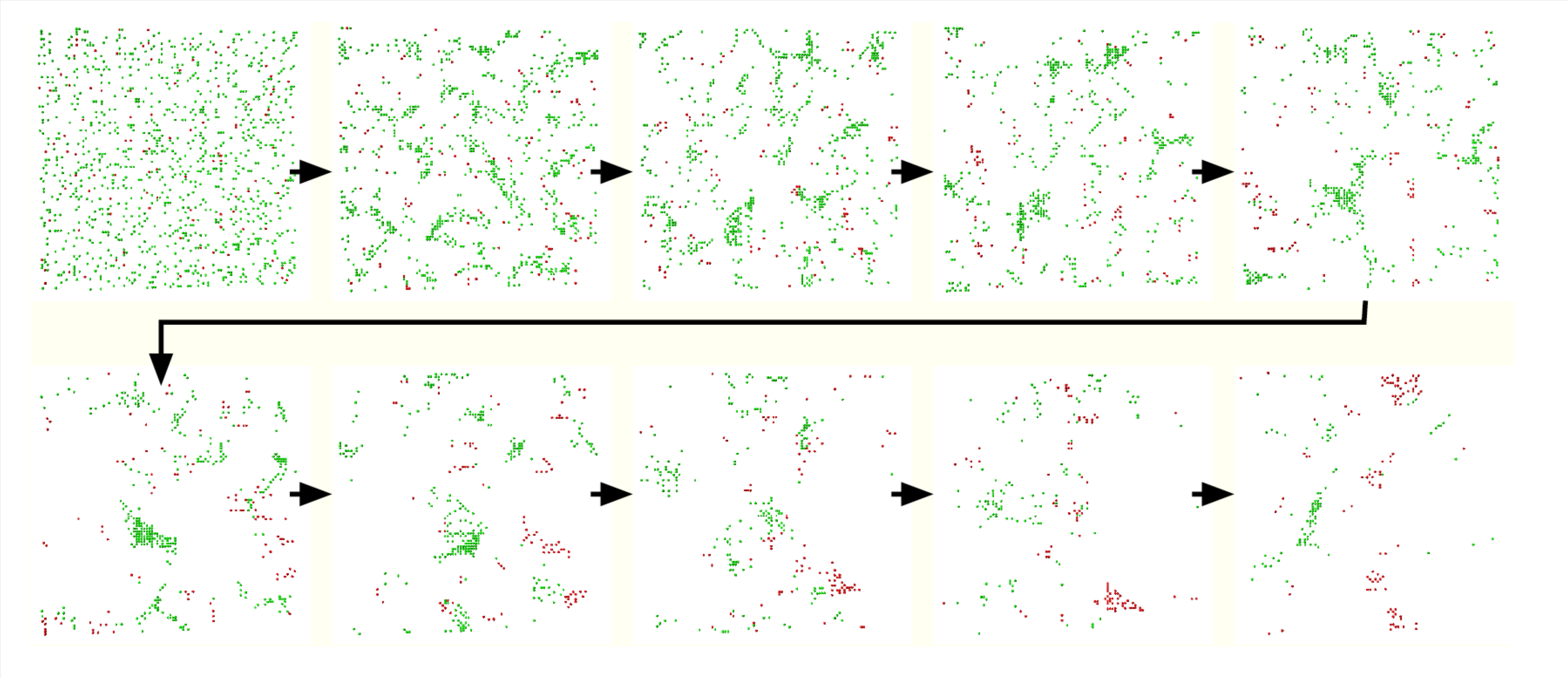
Typical Lifetime of Targets: $\tau = \Sigma t(N_T^t - N_T^{t-1})/N_T^0$

Cost: $c = N_C T / N_T^0$

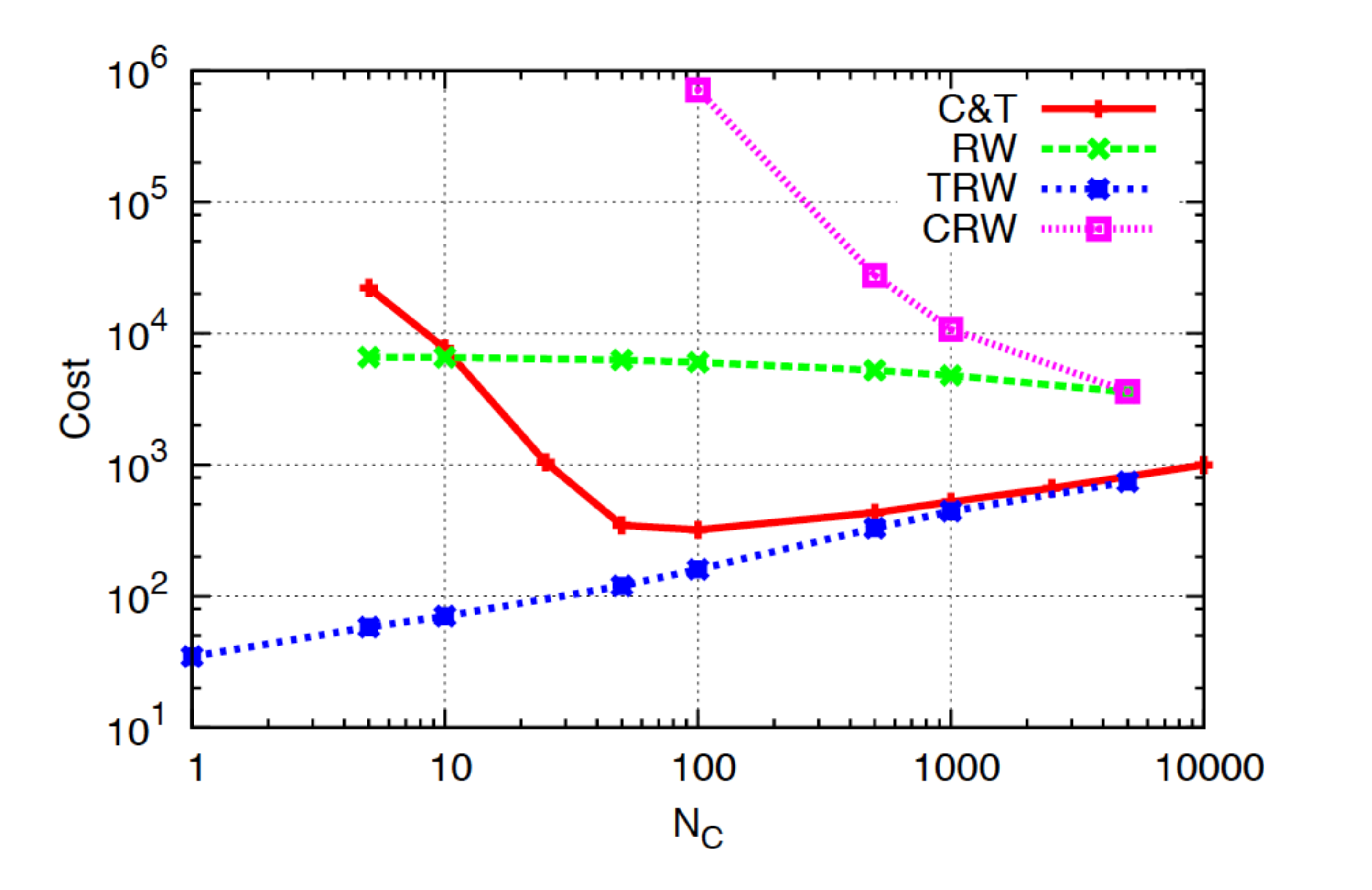
Simulation Results 2



$N_C = 100$ and $N_T^0 = 1000$

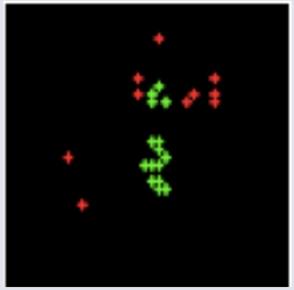


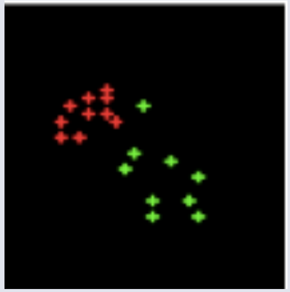
Simulation Results 3



$N_T^0 = 10$

Pattern Classification and Quantification





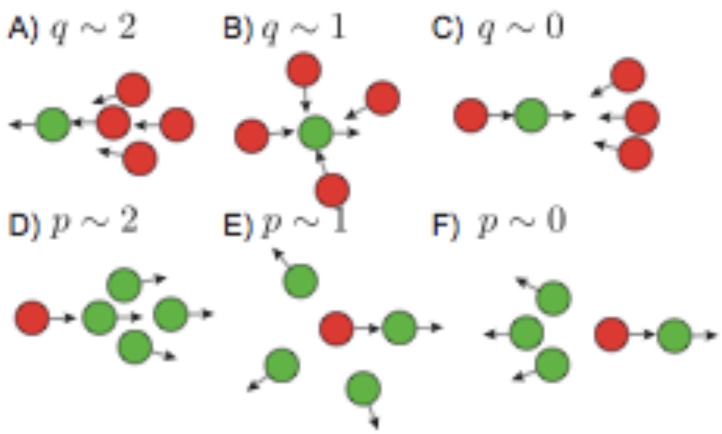


FIG. 2. Chasing and escaping processes characterized by parameters q and p. Pattern A, B, and C represent motion of chasers around an escapee with corresponding values of q. In contrast, pattern D, E, and F represent motion of escapees around a chaser with corresponding values of p.

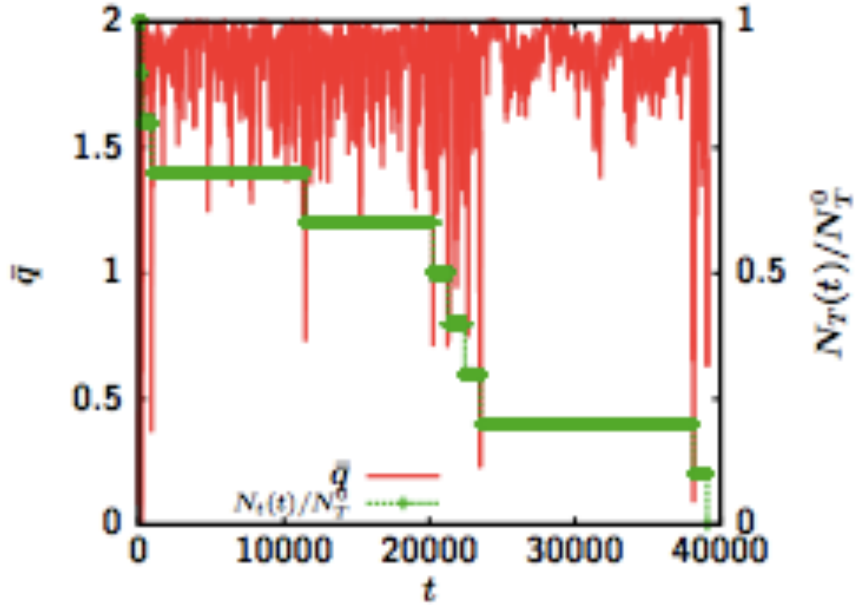
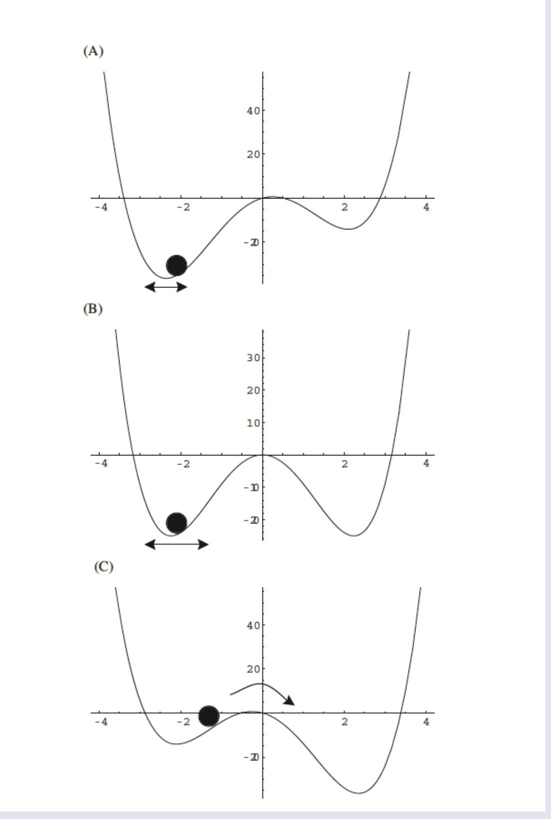


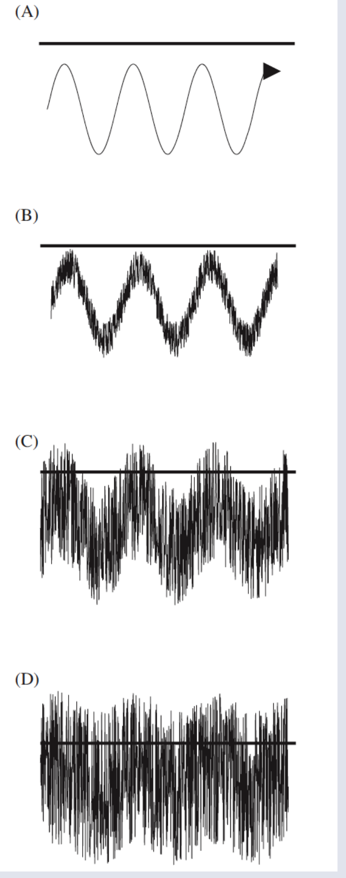
FIG. 5. Time evolution of $\bar{q}$ and $N_T(t)/N_T^0$ with $N_T^0 = 10$ and $N_C = 5$.

Stochastic Resonance

$$\frac{d}{dt}x(t) = -\frac{d}{dx}V(x) + A\cos(\omega t + \phi) + \xi(t)$$

$$V(x) = -\frac{a}{2}x^2 + \frac{b}{4}x^4$$





Model Extension 2: Making Errors in Steps

Stochastically makes errors in making steps to and from opponents

Hopping to neighboring site is now probabilistic with a "temperature":

Distance decrease $\Delta l_i = -1$

Distance increase $\Delta l_i = 1$

Chasers: $p_i^C = \exp(-\Delta l_i/T_f)/\Sigma_i \exp(-\Delta l_i/T_f)$

Targets: $p_i^T = \exp(\Delta l_i/T_f)/\Sigma_i \exp(\Delta l_i/T_f)$

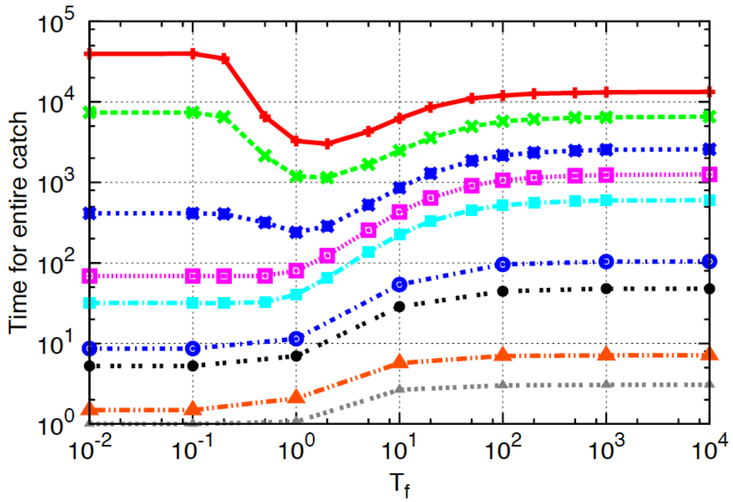


Figure 12. Time for entire catch as function of temperature T_f. Lines from the above to bottom are for $N_C = 5, 10, 25, 50, 100, 500, 1000, 5000, 9990$. For all lines, we fix $N_T^0 = 10$.

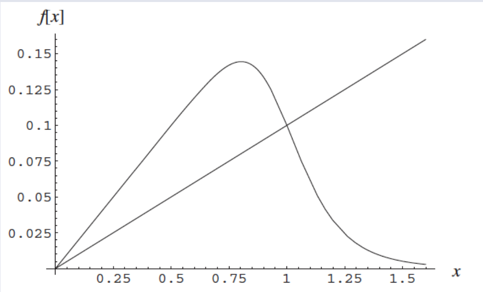
Delay Differential Equations

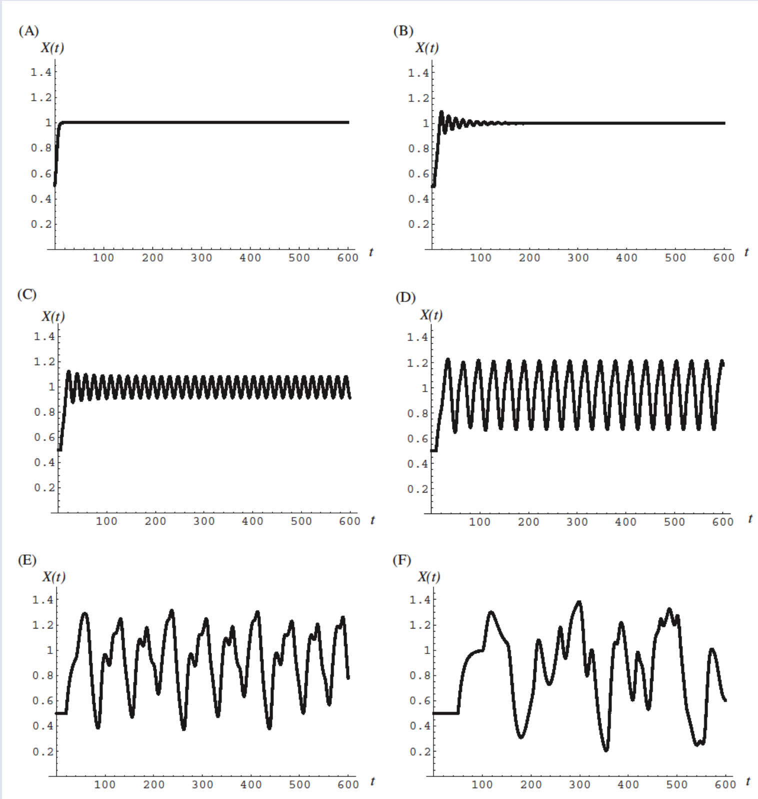
Typical Delayed Dynamics

$$\frac{1}{\alpha}\frac{d}{dt}X(t) + X(t) = f[X(t-\tau)]$$

* Mackey-Glass Model

$$\frac{d}{dt}X(t) + \alpha X(t) = \frac{\beta X(t-\tau)}{1+\{X(t-\tau)\}^n}$$

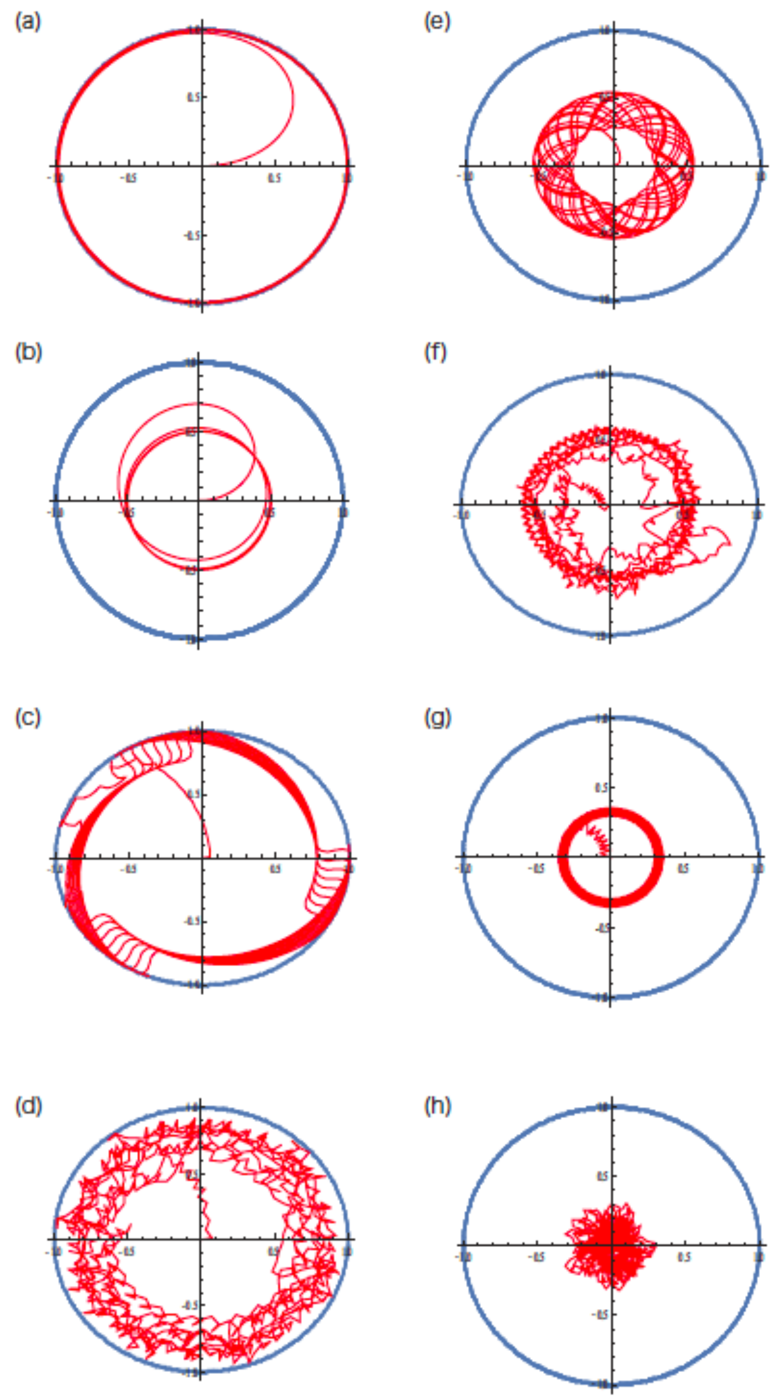




Analytically Unsolved!

Delays in Chase and Escape

- **Distant Dependent Delay**
- **The chaser points to the evader's past position**



Chases and Escapes and Optimization Problems:

（arXive1412.2114）

Chases and Escapes over "Energy Landscapes" of optimization problems

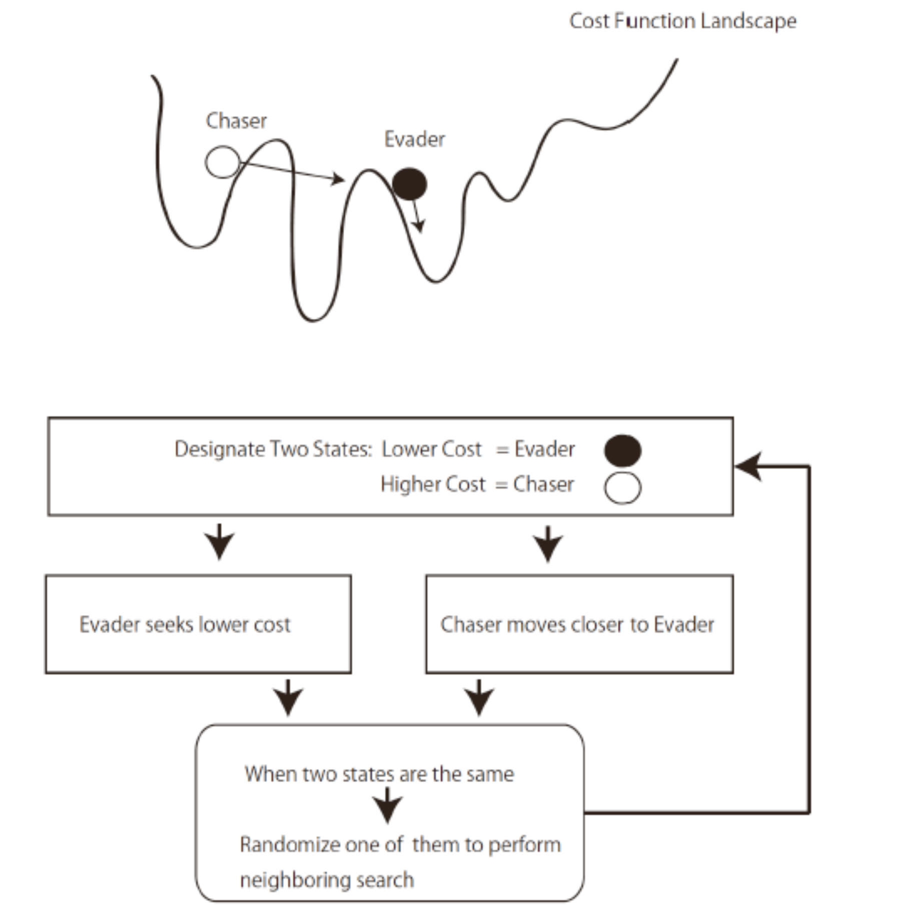


Fig. 2. Overall Design of the Chase and Escape mechanism for optimization problems.

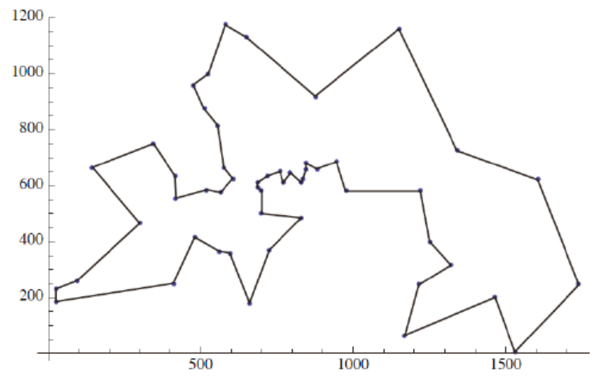


Fig. 3. Traveling Salesman Problem with 52 cities. The line is the shortest path. This best path length is approximately 7,544.

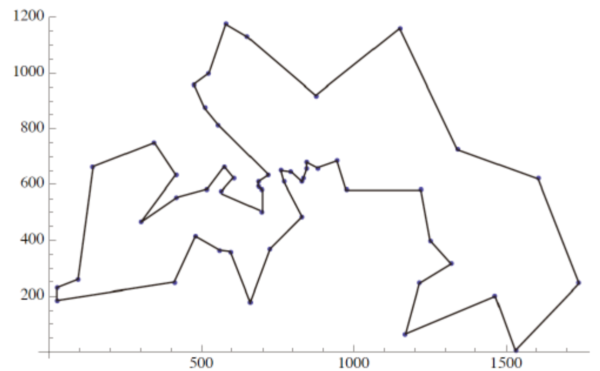


Fig. 4. An example of path for the Traveling Salesman Problem with 52 cities with our algorithm using chase and escape. This path length is approximately 7,940.

Discussion + Extra

Further work

- **Interaction among peers**
- **Off-Lattice Models**
- **Analytical Study**
- **Applications both in Physical and Cyber Space**