Traveling spots in an excitable medium and ventricular fibrillation

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Contents

- Motivations
- Traveling spots
  - Singular limit problem
  - Existence of traveling spot

Joint work with Y.-Y. Chen and Y. Kohsaka

- Application to ventricular fibrillation
  - Influence of obstacles
  - Tip formation

Joint work with M. Kaihara, T. Shimizu, N. J. Suematsu

Collaborators

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Y. Kohsaka (Muroran Institude of Technology)
M. Kaihara (Meiji University)
T. Shimizu (Meiji University)
N. J. Suematsu (Meiji University)

Patterns in an excitable medium

- To capture a solution of PDE (especially RD) in multi-dimensional space means to characterize its pattern
- There are many results for 1D
- Bifurcation of traveling spots

  Ohta, Mimura, Kobayashi (1989): radially symmetric SS

  Krischer, Mikhailov (1994): Bifurcation of TS from SS

  Or-Guil, Bode, Schenk, Purwins (1998):
  3 components system

  Pismen (2001): TS with sharp interface and bifurcation

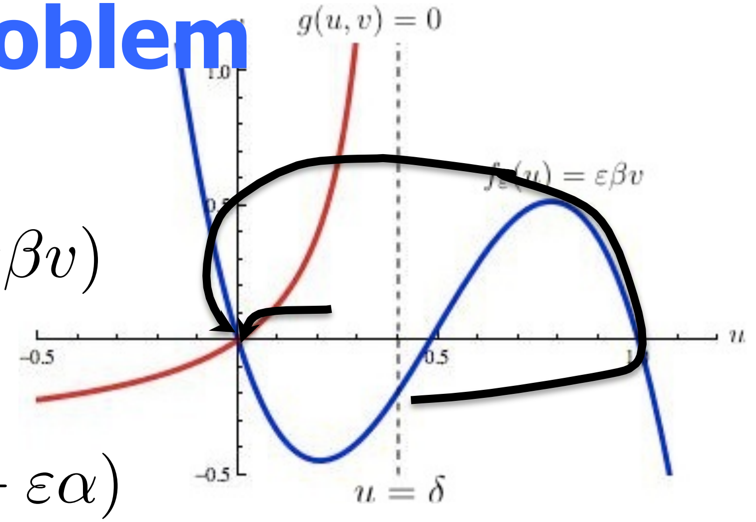
  Hagberg, Meron (2003), Nishiura, Teremoto, Ueda (2005)

- Photosensitive BZ reaction

  Mihaliuk, Sakurai, Chirila, Showalter (2001)
  Sakurai-Mihaliuk-Chirila-Showalter (2002, SCIENCE)
  Zykov, Showalter (2005): wave front interaction model
  in an excitable media
  Guo, Ninomiya, Tsai (2010)

As the first step, we need to get more information of non-radial TS.

FitzHugh-Nagumo equation and its singular limit problem



- FitzHugh-Nagumo equation

$$\begin{cases} u_t &= \Delta u + \dfrac{1}{\varepsilon^2}(f_\varepsilon(u) - \varepsilon\beta v) \\ v_t &= g(u,v) \end{cases}$$

where $$f_\varepsilon(u) = u(1-u)(u - \frac{1}{2} + \varepsilon\alpha)$$

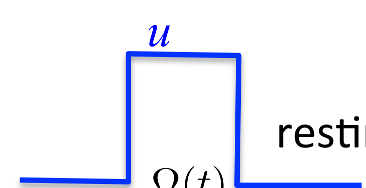
$$g(u,v) = g_1 u - \frac{g_2 v}{g_0 + g_3 v} \qquad \delta := \frac{g_2}{g_1 g_3} \le \frac{1}{2},\ g_i > 0$$

(ODE) $$\begin{cases} u_t &= \dfrac{1}{\varepsilon^2}(f_\varepsilon(u) - \varepsilon\beta v) \\ v_t &= g(u,v) \end{cases}$$

Excitable system (mono-stable)

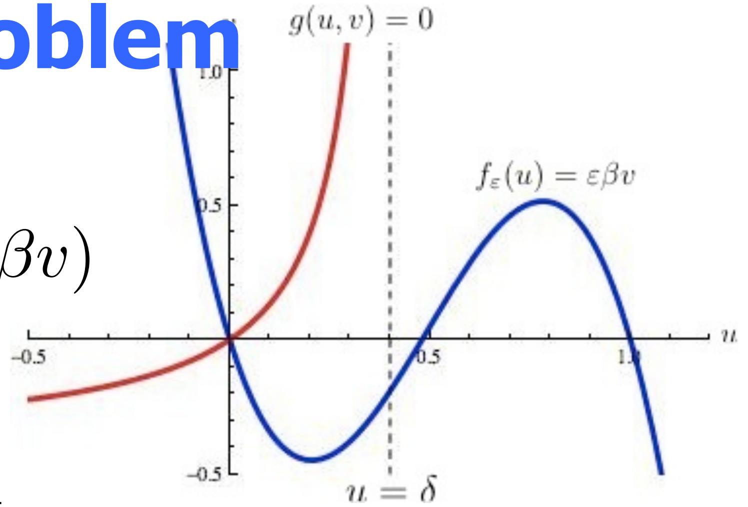
$$\begin{cases} \varepsilon^2 u_t &= \varepsilon^2 \Delta u + (f_\varepsilon(u) - \varepsilon\beta v) \\ v_t &= g(u,v) \end{cases}$$

Interface appears



$$f_0(u) = 0 \longrightarrow u = 0, \frac{1}{2}, 1$$

FitzHugh-Nagumo equation and its singular limit problem



- FitzHugh-Nagumo equation

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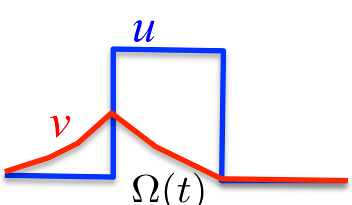
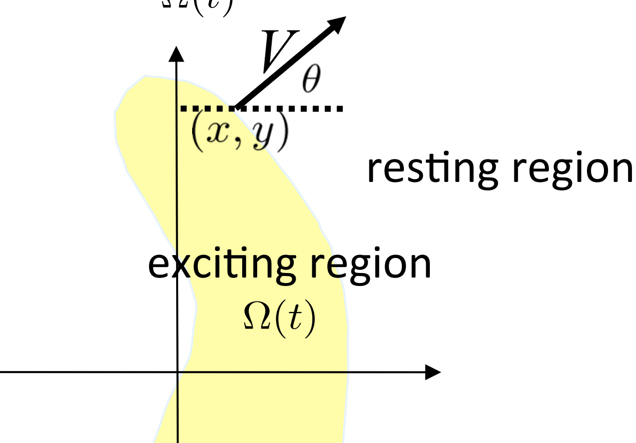


$$V = C(v) - \kappa$$
$$v_t = g(\chi_{\Omega(t)}, v)$$

V: normal velocity
h: a root of v=$f(u)$
κ :Curvature

χ: characteristic function



where $C(v) = a - bv$

$$a = \sqrt{2}\alpha, b = 6\sqrt{2}\beta$$

Planar waves with speed a

Theorem (Chen-Kohsaka-Ninomiya 2013)

$$V = a - bv - \kappa$$
$$v_t = g(\chi_{\Omega(t)}, v)$$

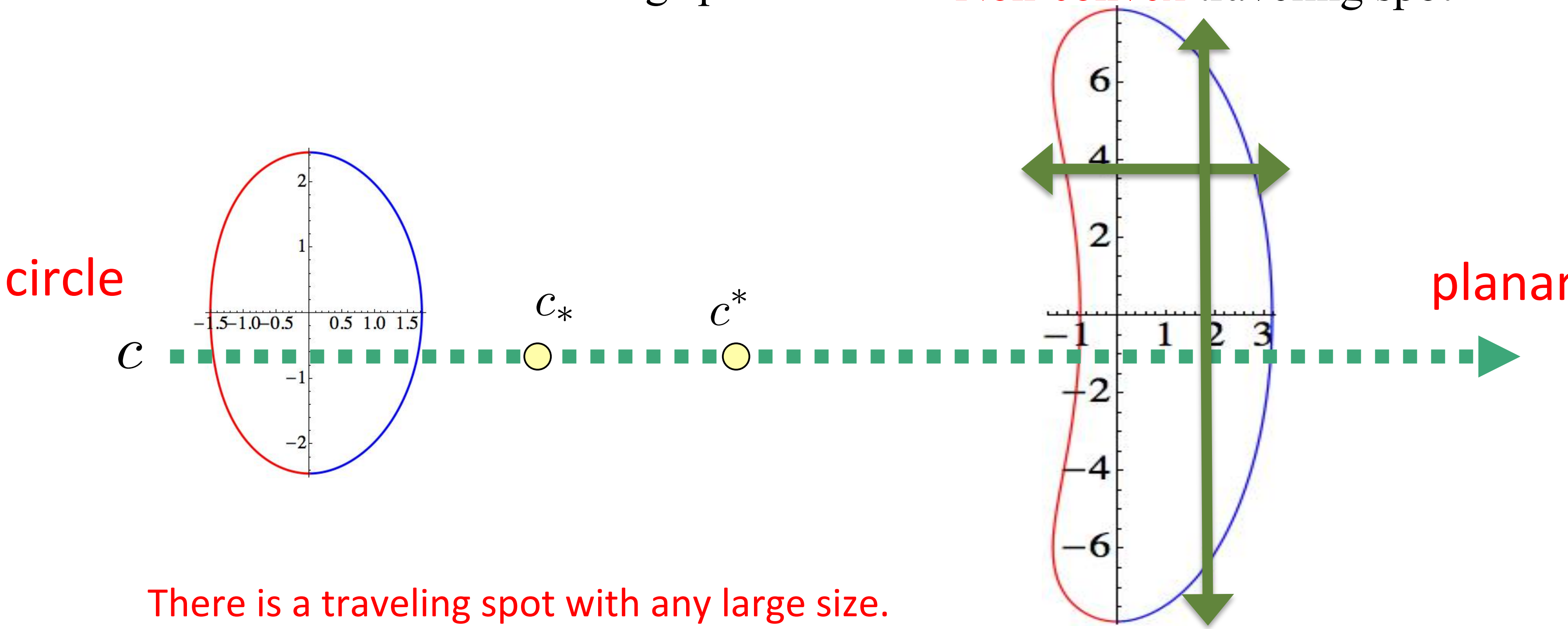
For any $0<c<a$, there are b and a traveling spot (Ω, v) with speed c such that

$$\lim_{x^2+y^2\to\infty} v(x,y,0) = 0, \quad v(x,y,0) = 0 \text{ if } |y| \geq Y_M.$$

Moreover,

(i) $0 \leq c \leq c_*$ There is a convex traveling spot

(ii) $c^* < c < a$ Non-convex traveling spot



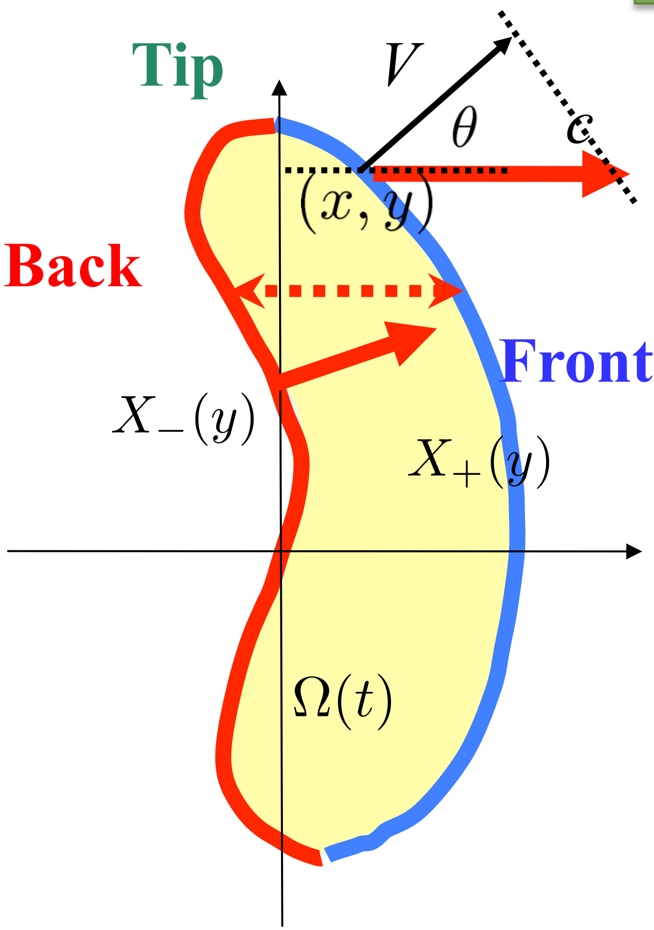
Outine of proof

$$V = a - bv - \kappa$$
$$v_t = g(\chi_{\Omega(t)}, v)$$

Traveling wave solution

$$\Omega(t) = \{(x + ct, y) \mid (x, y) \in \Omega(0)\}$$
$$v(x, y, t) = v(x - ct, y)$$

$$c\cos\theta = a - bv - \kappa$$
$$-cv_x = g(\chi_{\Omega(0)}, v)$$



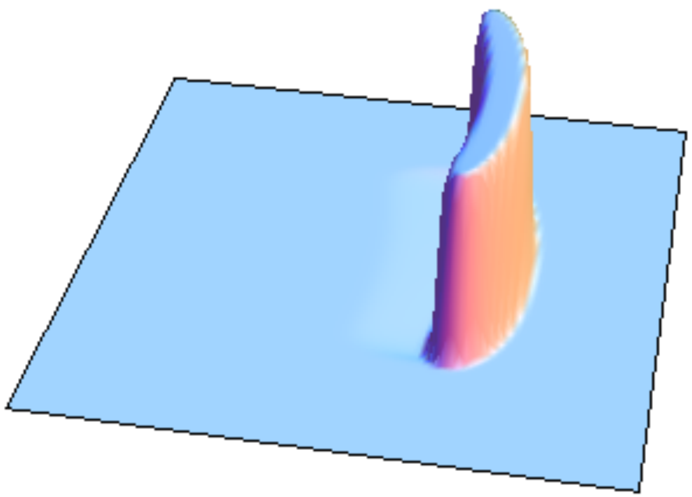
We denote the interface where V>0 (resp. V<0, V=0) by the front $x = X_+(y)$ (resp. back $x = X_-(y)$, tip)

Due to the effect of v, V is negative.

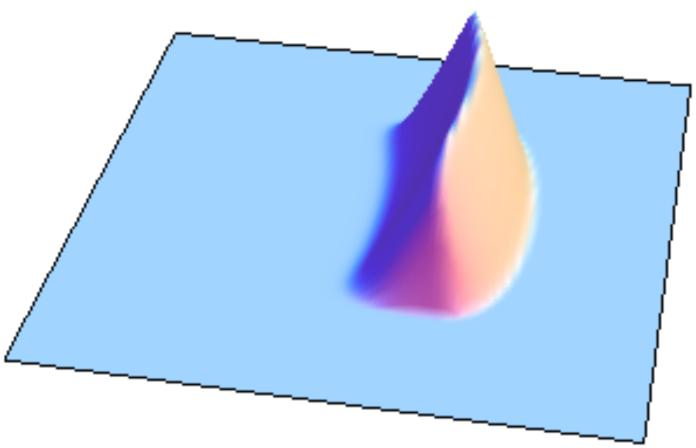
$$\begin{cases} \dfrac{dx_\pm}{ds} = -\sin\theta_\pm \\ \dfrac{dy_\pm}{ds} = \cos\theta_\pm \\ \dfrac{d\theta_\pm}{ds} = \kappa_\pm \quad = a - bv_\pm - c\cos\theta_\pm \end{cases}$$

Numerical solutions of FitzHugh-Nagumo equation

$$\begin{cases} u_t & = & \Delta u + \frac{1}{\varepsilon^2}(f_\varepsilon(u) - \varepsilon\beta v) \\ v_t & = & g(u,v) \end{cases}$$



Profile of u



Profile of v

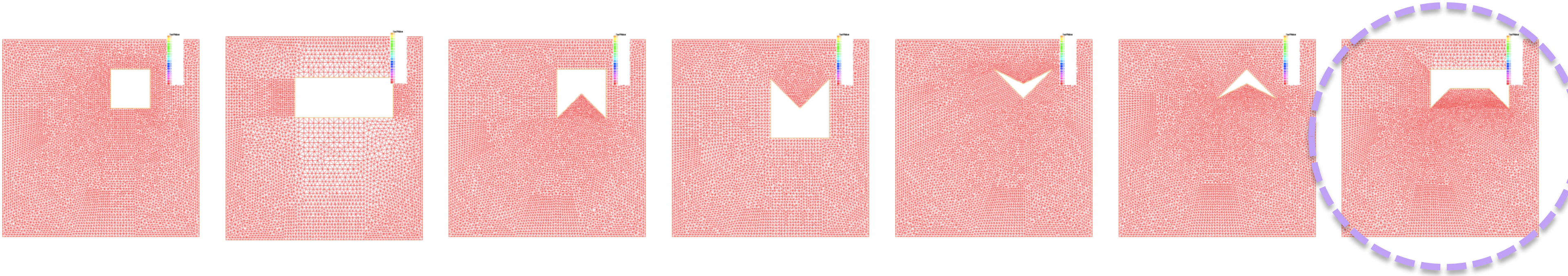
Application to ventricular fibrillation (VF)

心室細動

$$\begin{cases} u_t = \Delta u + u - u^3 - v \\ v_t = u - av + b \end{cases}$$

Wavebreaks may take place by an obstacle

- Jalife et al (1998)
- Tanaka (2008)

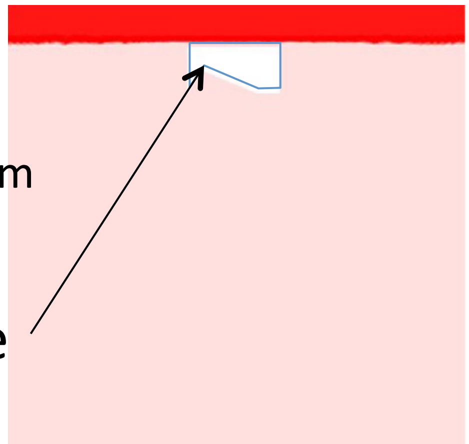


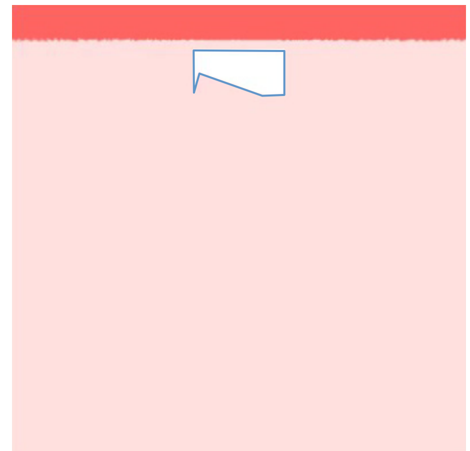
Formation of wavebreaks

- Shimizu, Kaihara, Suematsu and N. (2013)

Heart as an excitable medium

Obstacle





Wave moves downward

Fibrillation occurs by obstacle

Using numerical calculation, we can characterize the relationship between the shape of domain and the tip formation.