An Economic Analysis of Joint Products under Demand Uncertainty

Mahito Okura
Doshisha Women's College of Liberal Arts
December 8, 2018
The Workshop on Insurance and Finance
(jointly with Meiji Institute for Advanced Study of Mathematical Sciences (MIMS))

1. Introduction: demand information

- Demand information is an important factor in determining production quantities of firms.
- In some cases, some or all firms do not have demand information for their market.
- In such cases, firms only use information about “expected” demand and there may be a gap between expected and actual demand levels.
- Asymmetric information problem might be realized.

1. Introduction: previous studies

- Microeconomics of uncertainty: Basar and Ho (1974), Ponssard (1979),Vives (1984), Sakai and Yoshizumi (1991), Sakai and Sasaki (1992), Sakai (1993), Asplund (2002), Chokler et al. (2006), Arya et al. (2013), Afeche et al. (2014), Okura (2014)
- Flexible technology under demand uncertainty: Goyal and Netessaine (2011), Boyabatli (2015), Boyabatli et al. (2015)

1. Introduction: joint products

- Joint products: Products indivisibly produced from a single input.
- Example: petroleum products are produced simultaneously, and the production quantity ratio of these products is exogenously decided.
  - Gasoline: 27.3%
  - Naphtha: 10.2%
  - Jet fuel: 7.7%
  - Kerosene: 8.8%
  - Light oil: 21.6%
  - Heavy oil: 18.4%

  (in fiscal 2013. Source: website of the Petroleum Association of Japan)

1. Introduction: research purposes

- This study analyzes the market of joint products under demand uncertainty.
- It is related to economic models of uncertainty and flexible technology, but…
  - the market of joint products has not been discussed in economics of uncertainty.
  - the main focus of the studies in flexible technology is on the characteristics of or differences among technologies, while demand uncertainty is considered as a market condition.
  - In contrast, this study discusses the effect of demand uncertainty and asymmetric information, and investigates how results differ under different information structures, while treating technologies for producing multiple products are an exogenous variable.

2. Model setting: notation

- Two firms (firm A and B)
- Two joint products (product 1 and 2)
- Unit cost of the input : $c > 0$
- Production quantity of product 1: $q_1^i = \gamma x^i$ where $i \in \{A, B\}$
- Production quantity of product 2: $q_2^i = (1-\gamma)x^i$
  - $\gamma \in [0,1]$: product 1's ratio (exogenous variable)
  - $1-\gamma$: product 2' ratio
  - "one-input, one-output" situation: $\gamma = 0$ and $\gamma = 1$
- x^i: Amount of input of firm i

2. Model setting: demand function and information structure

- Demand function: $p_j = a_j - b_j\left(q_j^A + q_j^B\right)$ where $j \in \{1,2\}$
- The firm might not have demand information for the market j, represented by a_j in the above demand function.
- $\eta_j^i \in \{0,1\}$: Firm i's demand information situation in market j
  - $\eta_j^i = 0$: firm i does not have demand information in market j
  - $\eta_j^i = 1$: firm i has full demand information in market j
- Information structure: $\eta \equiv \{\eta_1^A, \eta_2^A, \eta_1^B, \eta_2^B\}$

2. Model setting: demand information and profit function

- Demand information in the two markets (a_1, a_2) has a joint probability distribution and is normally distributed as (μ_1, μ_2), (σ_1^2, σ_2^2), and $\rho \in [-1,1]$.
  - $\mu_j \equiv E[a_j]$
  - $\sigma_j^2 \equiv E\left[(a_j - \mu_j)^2\right]$
  - ρ : Correlation coefficient of the two demand information

- The profit function of each firm: $\Pi^i = p_1 q_1^i + p_2 q_2^i - cx^i$
- Each firm competes on the basis of production quantity and determines the amount of inputs for their products to maximize their own expected profit.

3. Information structures

- The number of information structures: 16
- The following 7 situations can arise:
  - Situation 1: no firm has any demand information ($\eta = \{0,0,0,0\}$).
  - Situation 2: one firm has demand information in either market, but the other firm does not ($\eta = \{1,0,0,0\}$, $\eta = \{0,1,0,0\}$, $\eta = \{0,0,1,0\}$, $\eta = \{0,0,0,1\}$).
  - Situation 3: each firm has demand information in either market, but demand information of each firm is different ($\eta = \{1,0,0,1\}, \eta = \{0,1,1,0\}$).

3. Information structures

- Situation 4: each firm has demand information in either market and demand information of each firm is the same ($\eta = \{1,0,1,0\}, \eta = \{0,1,0,1\}$).
- Situation 5: one firm has demand information in both markets (that is, perfect information), but the other firm has no demand information ($\eta = \{1,1,0,0\}, \eta = \{0,0,1,1\}$).
- Situation 6: one firm has demand information for both markets, but the other firm has demand information in one market ($\eta = \{1,1,1,0\}$, $\eta = \{1,1,0,1\}$, $\eta = \{1,0,1,1\}$, $\eta = \{0,1,1,1\}$).
- Situation 7: both firms have demand information in both markets, that is, no demand uncertainty exists ($\eta = \{1,1,1,1\}$).

4. Deriving equilibrium expected profit

- The equilibrium expected profit is derived for each situation.
- The results of the computations in all seven situations are summarized in Table 1 (see separated document).
  - $E\left[\Pi^i\left(\eta_1^A, \eta_2^A, \eta_1^B, \eta_2^B\right)\right]$: the equilibrium expected profit of firm i when the information structure is $\eta = \{\eta_1^A, \eta_2^A, \eta_1^B, \eta_2^B\}$.

5. Investigation of the equilibrium expected profit in each information structure: Result 1

- $E[\Pi^A(0,0,0,0)] = E[\Pi^A(0,0,1,0)] = E[\Pi^A(0,0,0,1)] = E[\Pi^A(0,0,1,1)]$
- $E[\Pi^A(1,0,0,1)] = E[\Pi^A(1,0,1,0)] = E[\Pi^A(1,0,1,1)]$
- $E[\Pi^A(0,1,0,1)] = E[\Pi^A(0,1,1,0)] = E[\Pi^A(0,1,1,1)]$

5. Investigation of the equilibrium expected profit in each information structure: Result 1

Result 1:

- The equilibrium expected profit of firm A when it has no demand information or demand information for one market are the same, regardless of the information structure of firm B.

5. Investigation of the equilibrium expected profit in each information structure: Result 2

- $E[\Pi^A(1,1,0,0)] > E[\Pi^A(1,1,1,0)]$
- $E[\Pi^A(1,1,0,0)] > E[\Pi^A(1,1,0,1)]$
- $E[\Pi^A(1,1,1,0)] \geq E[\Pi^A(1,1,1,1)]$
- $E[\Pi^A(1,1,0,1)] \geq E[\Pi^A(1,1,1,1)]$
- If $\gamma^2\sigma_1^2 > (<)(1-\gamma)^2\sigma_2^2$ (where $-1 < \rho < 1$), then $E[\Pi^A(1,1,0,1)] > (<)E[\Pi^A(1,1,1,0)]$

5. Investigation of the equilibrium expected profit in each information structure: Result 2

Result 2:

- The equilibrium expected profit of firm A when it has demand information for both markets depend on the information structure of firm B. If firm B has no (either, both) demand information, the equilibrium expected profit of firm A is high (middle, low).
- If the variance of demand information in market 1 is higher (lower) than that in market 2, the equilibrium expected profit of firm A, when it has demand information for both markets but firm B has demand information for only market 1, is smaller (larger) than when firm B has demand information for only market 2.

5. Investigation of the equilibrium expected profit in each information structure: Result 3

- If $\gamma^2\sigma_1^2 > (<)(1-\gamma)^2\sigma_2^2$ (where $-1 < \rho < 1$), then
  - $E[\Pi^A(1,0,0,1)] > (<)E[\Pi^A(0,1,0,1)]$
  - $E[\Pi^A(1,0,0,0)] > (<)E[\Pi^A(0,1,0,0)]$

5. Investigation of the equilibrium expected profit in each information structure: Result 3

Result 3:

If the variance of demand information in market 1 is higher (lower) than that in market 2, the equilibrium expected profit of firm A, which has demand information in market 1, is larger (smaller) than if it has demand information in market 2, when firm B has either no demand information or demand information for one market.

5. Investigation of the equilibrium expected profit in each information structure: Result 4

- If $\rho \in \left(\underline{\rho}, \bar{\rho}\right)$, $E[\Pi^A(1,0,0,0)] < E[\Pi^A(1,1,0,1)]$ is realized
  - $\underline{\rho}$ and $\bar{\rho}$: values for which the result of the following equation is zero
    - $E[\Pi^A(1,0,0,0)] - E[\Pi^A(1,1,0,1)] = \frac{\{5\gamma^2\sigma_1^2+9(1-\gamma)^2\sigma_2^2\}\rho^2+10(1-\gamma)\rho\sigma_1\sigma_2-4(1-\gamma)^2\sigma_2^2}{36(\gamma^2 b_1+(1-\gamma)^2 b_2)}$

5. Investigation of the equilibrium expected profit in each information structure: Result 4

Result 4:

Suppose that firm A has demand information in market 1 (market 2) and firm B has no demand information for this market. Where the absolute value of the correlation coefficient is relatively large (small), the equilibrium expected profit of firm A decreases (increases) when both firms have demand information in market 2 (market 1).

5. Investigation of the equilibrium expected profit in each information structure: Result 5

- In the case of $\rho = 1$ or $\rho = -1$:
  - $E[\Pi^A(1,1,0,0)] = E[\Pi^A(1,0,0,0)] = E[\Pi^A(0,1,0,0)]$
  - $E[\Pi^A(1,1,0,1)] = E[\Pi^A(1,1,1,0)] = E[\Pi^A(1,1,1,1)] = E[\Pi^A(1,0,0,1)] = E[\Pi^A(1,0,1,0)] = E[\Pi^A(1,0,1,1)] = E[\Pi^A(0,1,1,1)] = E[\Pi^A(0,1,1,0)] = E[\Pi^A(0,1,0,1)]$
  - $E[\Pi^A(0,0,0,0)] = E[\Pi^A(0,0,0,1)] = E[\Pi^A(0,0,1,0)] = E[\Pi^A(0,0,1,1)]$

- Then, $E[\Pi^A(1,1,0,0)] > E[\Pi^A(1,1,0,1)] > E[\Pi^A(0,0,0,0)]$

5. Investigation of the equilibrium expected profit in each information structure: Result 5

Result 5:

Where demand information in two markets are perfectly correlated, equilibrium expected profit will be equal in each of the following three cases: (1) firm A has demand information in either or both markets, firm B has no demand information; (2) both firms have demand information in either or both markets, (3) firm A has no demand information, firm B has demand information in either or both markets. Results for these information structures are therefore the same in a single market.

5. Investigation of the equilibrium expected profit in each information structure: Result 6

- $f_{00} \equiv E[\Pi^A(1,0,0,0)] - E[\Pi^A(0,0,0,0)] = \frac{\{\gamma\sigma_1+(1-\gamma)\rho\sigma_2\}^2}{4(\gamma^2 b_1+(1-\gamma)^2 b_2)} > 0$
- $f_{01} \equiv E[\Pi^A(1,0,0,1)] - E[\Pi^A(0,0,0,1)] = \frac{\{\gamma\sigma_1+(1-\gamma)\rho\sigma_2\}^2}{9(\gamma^2 b_1+(1-\gamma)^2 b_2)} > 0$
- $f_{10} \equiv E[\Pi^A(1,0,1,0)] - E[\Pi^A(0,0,1,0)] = \frac{\{\gamma\sigma_1+(1-\gamma)\rho\sigma_2\}^2}{9(\gamma^2 b_1+(1-\gamma)^2 b_2)} > 0$
- $f_{11} \equiv E[\Pi^A(1,0,1,1)] - E[\Pi^A(0,0,1,1)] = \frac{\{\gamma\sigma_1+(1-\gamma)\rho\sigma_2\}^2}{9(\gamma^2 b_1+(1-\gamma)^2 b_2)} > 0$

- Then, $f_{00} > f_{01} = f_{10} = f_{11}$

5. Investigation of the equilibrium expected profit in each information structure: Result 6

Result 6:

Where firm A has no demand information, its equilibrium expected profit always increases when it obtains demand information for either market. The increase in the equilibrium expected profit of firm A when firm B has no demand information is larger than when firm B has demand information for either or both markets. Furthermore, the increase in the equilibrium expected profit of firm A when firm B has demand information for one market is the same as when firm B has demand information for both markets.

5. Investigation of the equilibrium expected profit in each information structure: Result 7

- $g_{00} \equiv E[\Pi^A(1,1,0,0)] - E[\Pi^A(0,1,0,0)] = \frac{(1-\rho^2)\gamma^2\sigma_1^2}{4(\gamma^2 b_1+(1-\gamma)^2 b_2)} \geq 0$
- $g_{01} \equiv E[\Pi^A(1,1,0,1)] - E[\Pi^A(0,1,0,1)] = \frac{(1-\rho^2)\gamma^2\sigma_1^2}{4(\gamma^2 b_1+(1-\gamma)^2 b_2)} \geq 0$
- $g_{10} \equiv E[\Pi^A(1,1,1,0)] - E[\Pi^A(0,1,1,0)] = \frac{(1-\rho^2)\{4\gamma^2\sigma_1^2+5(1-\gamma)^2\sigma_2^2\}}{36(\gamma^2 b_1+(1-\gamma)^2 b_2)} \geq 0$
- $g_{11} \equiv E[\Pi^A(1,1,1,1)] - E[\Pi^A(0,1,1,1)] = \frac{(1-\rho^2)\gamma^2\sigma_1^2}{9(\gamma^2 b_1+(1-\gamma)^2 b_2)} \geq 0$

- Then,
  - If $\gamma^2\sigma_1^2 > (1-\gamma)^2\sigma_2^2$, then $g_{00} = g_{01} \geq g_{10} \geq g_{11}$
  - If $\gamma^2\sigma_1^2 < (1-\gamma)^2\sigma_2^2$, then $g_{10} \geq g_{00} = g_{01} \geq g_{11}$

5. Investigation of the equilibrium expected profit in each information structure: Result 7

Result 7:

Where firm A has demand information in market 2, then obtains demand information in market 1, the equilibrium expected profit of firm A will always increase. Where the variance of demand information in market 1 is larger (smaller) than market 2 and firm B has no demand information or demand information in market 2 only, the increase in the equilibrium expected profit of firm A on obtaining demand information in market 1 is largest (second largest); when firm B has demand information in market 1 only, the increase is second largest (largest). When firm B has demand information in both markets, the increase in the equilibrium expected profit of firm A is lowest.

5. Investigation of the equilibrium expected profit in each information structure: Result 8

- Investigate the possibility of exchanging demand information between two firms:
  - Firm A has demand information in market 1, but not in market 2.
  - Firm B has demand information in market 2, but not in market 1.
  - Here, the information structure is $\eta = \{1,0,0,1\}$.
  - Each firm has two choices: “disclose own demand information” or “do not disclose own demand information.”

5. Investigation of the equilibrium expected profit in each information structure: Result 8

Firm A \ Firm B	Disclose	Not disclose
Disclose	$\eta = \{1,1,1,1\}$	$\eta = \{1,0,1,1\}$
Not disclose	$\eta = \{1,1,0,1\}$	$\eta = \{1,0,0,1\}$

Table 2: Firm A and B's choices and corresponding information structures

5. Investigation of the equilibrium expected profit in each information structure: Result 8

- $E[\Pi^A(1,1,0,1)] \geq E[\Pi^A(1,1,1,1)] \geq E[\Pi^A(1,0,0,1)] = E[\Pi^A(1,0,1,1)]$
- $E[\Pi^B(1,0,1,1)] \geq E[\Pi^B(1,1,1,1)] \geq E[\Pi^B(1,0,0,1)] = E[\Pi^B(1,1,0,1)]$
- $E\left[\Pi^i(1,1,1,1)\right] \geq E\left[\Pi^i(1,0,0,1)\right]$

- Nash equilibrium: both firms choose “Not disclose” (dominant strategy) and $\eta = \{1,0,0,1\}$ still holds.
- Outcome in Nash equilibrium is Pareto-inferior outcome.

5. Investigation of the equilibrium expected profit in each information structure: Result 8

Result 8:

Where firm A has demand information in market 1 (market 2) and firm B has demand information in market 2 (market 1), if both firms decide their disclosure policy independently, we would expect no exchange of demand information because a prisoners' dilemma situation is realized. Then, Pareto-inferior outcome cannot be improved.

6. Conclusion

- From the results, for example, we know
  - Policymakers must know the variances and correlation coefficients of demand information in two markets to realize an optimal outcome.
  - Some kind of coordination or incentive schemes are needed for realizing desirable outcomes through exchange of demand information. One solution is that, instead of each firm, industrial association investigates the demand information and discloses it to all member firms.