Mathematical model for spreading *Physarum*

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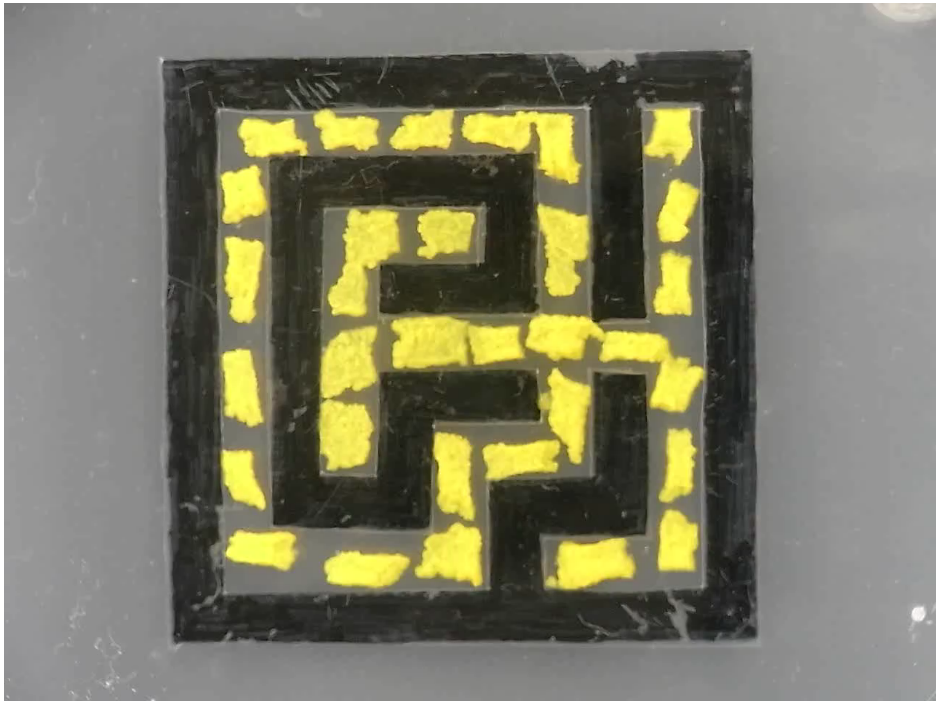
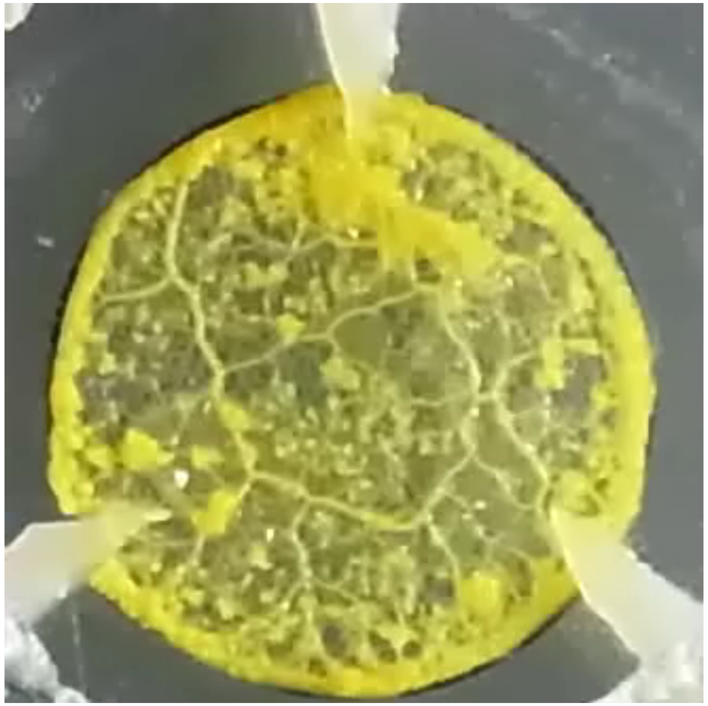
Plasmodium of true Slime mold

Unicellular organism containing multi nuclei

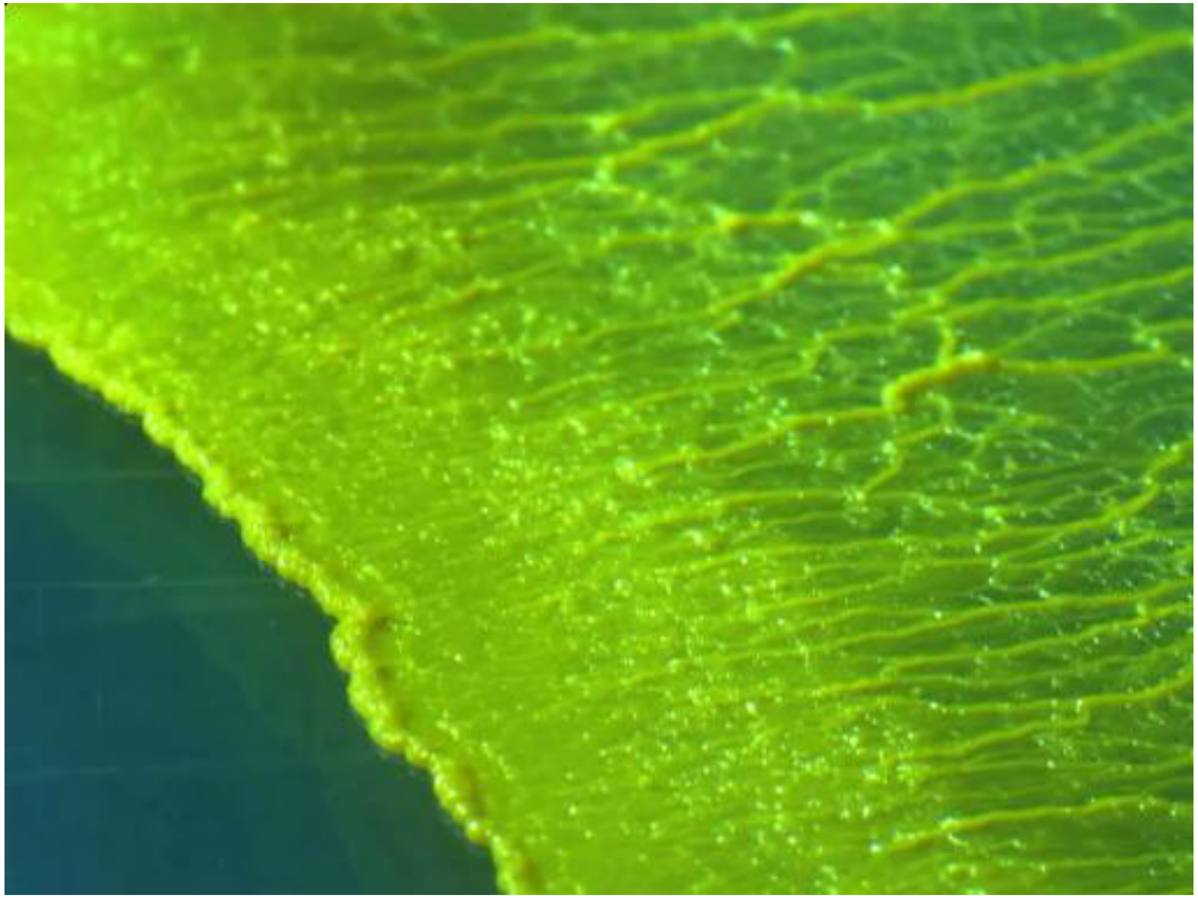




30mm

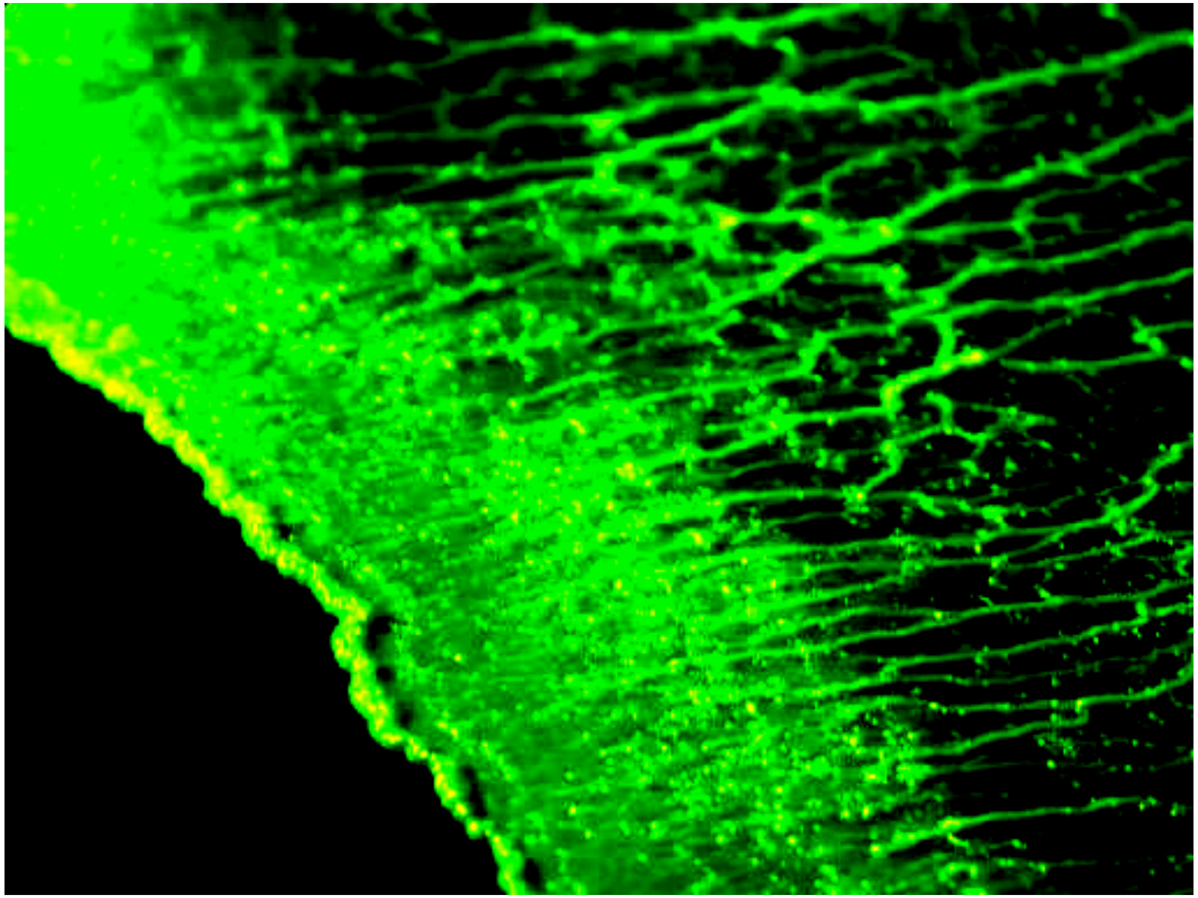
No nervous system.
Local information processing + Mechanics

Propagation of Slime mold



Thickness oscillation + Propagation

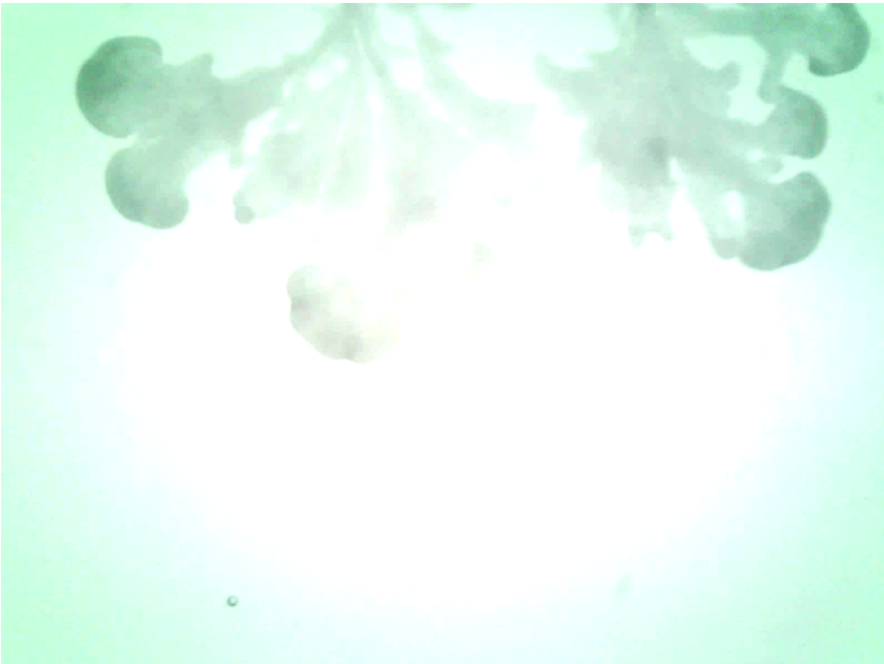
Oscillation of slime mold

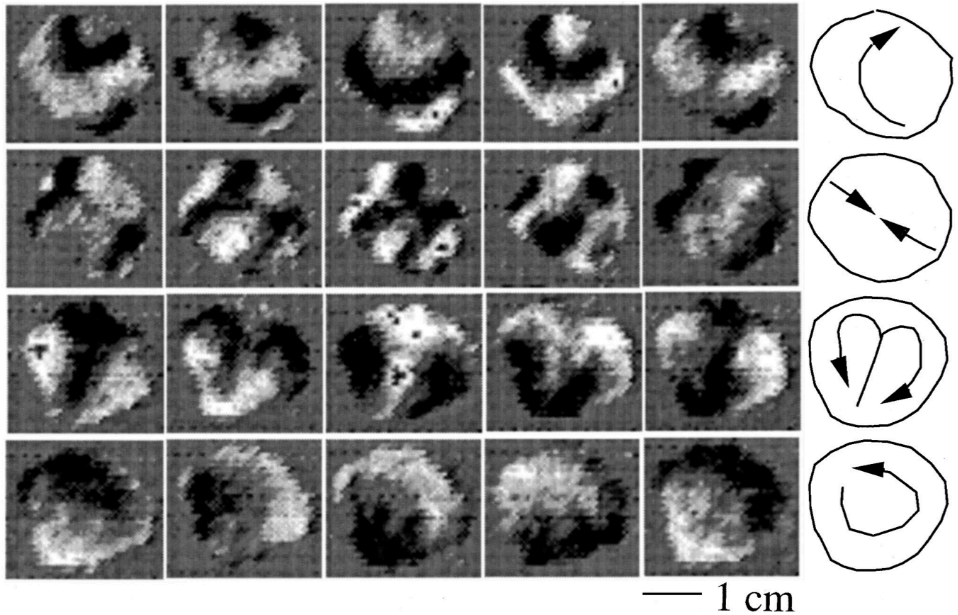


(x 60)

Thickness oscillation + Propagation

Thickness oscillation





T.Nakagaki, et. al. Biophys. Chem. (2000)

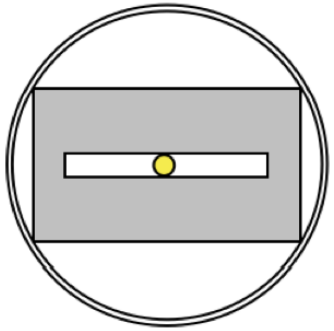
Mathematical models

A. Tero, R. Kobayashi and T. Nakagaki, Physica D (2005)

R. Kobayashi, A. Tero and T. Nakagaki, J. Theor. Biol. (2006)

K. Ueda, Phys. Rev. E. (2011)

Two typical locomotion in a lane



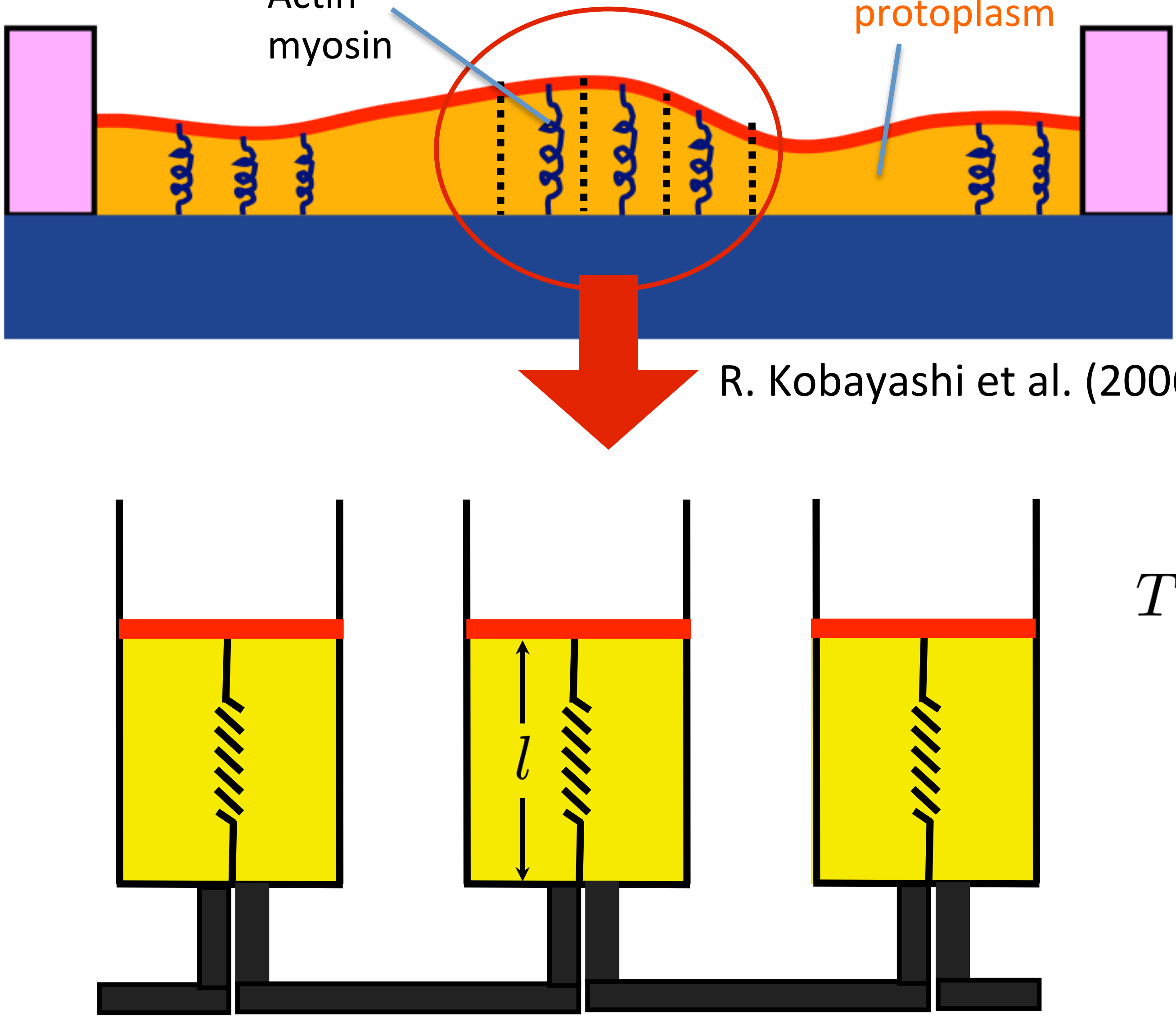
Moving both side

One direction moving + ?

Question:
How do they know the edge, and turn to the direction?

1. Oscillation model

Thickness oscillation



R. Kobayashi et al. (2006)

Spring coefficient

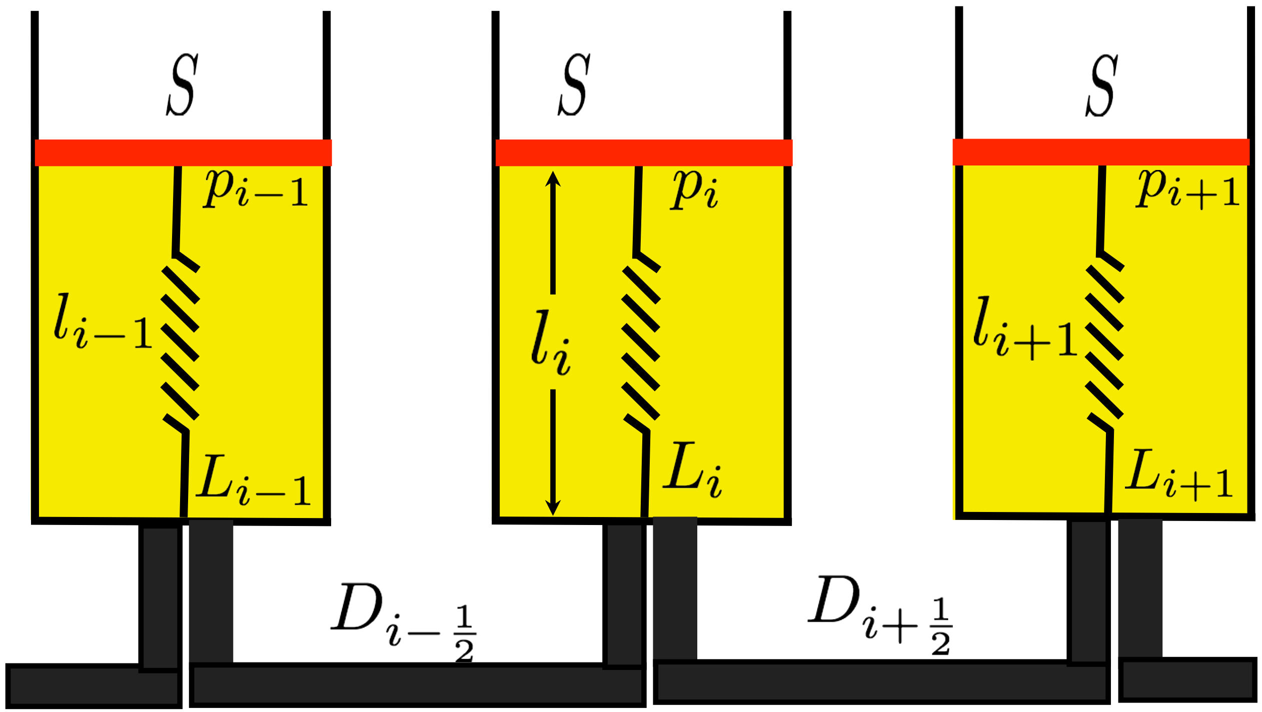
Natural length

$$T = \alpha(l - L)$$

$$= \frac{\kappa}{L}(l - L)$$

$$= \kappa(\frac{l}{L} - 1)$$

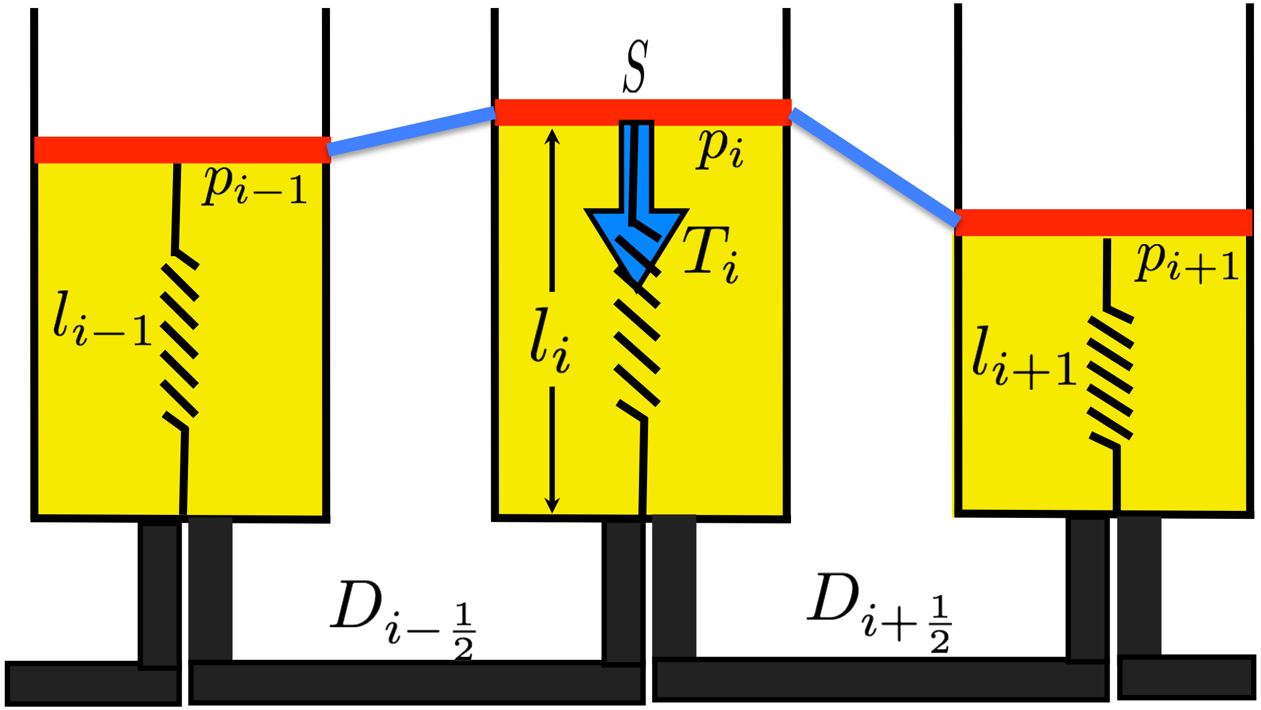
Thickness oscillation model



Spring tension: $T_i = \max\{\kappa_i \left(\frac{l_i}{L_i} - 1\right), 0\}$

Volume of protoplazm: $S\dot{l_i} = D_{i-\frac{1}{2}}(p_{i-1} - p_i) + D_{i+\frac{1}{2}}(p_{i+1} - p_i)$

Interaction between cylinder heads



$$Sp_i = T_i + \underline{k_{i+\frac{1}{2}}(l_i - l_{i+1}) + k_{i-\frac{1}{2}}(l_i - l_{i-1})}$$

Elastic film connecting among cylinder heads

Dynamics of active spring

Natural length of spring i:

$$L_i = \bar{L}_i(1 + a\sin\phi_i)$$

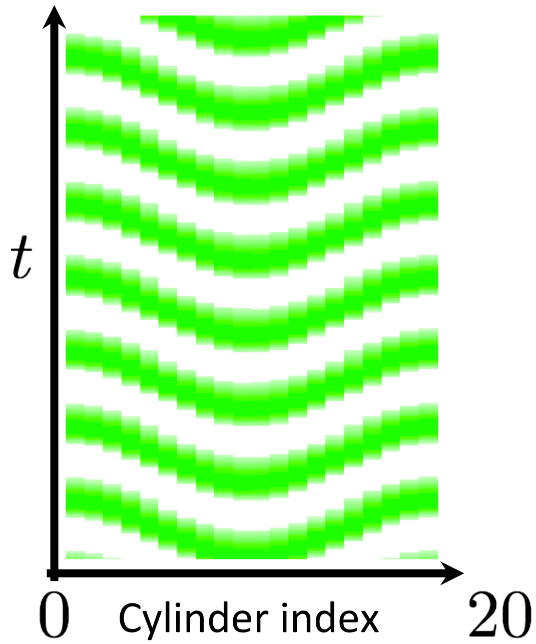
Phase dynamics:

$$\dot{\phi}_i = \omega + \underline{T_i\Gamma(\phi_i, l_i, \bar{L}_i)}$$

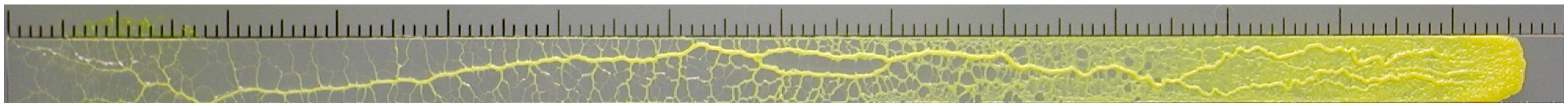
Phase modification term

$$\Gamma(\phi_i, l_i, \bar{L}_i) = \frac{\sigma a l_i \kappa_i}{\bar{L}(1 + a\sin\phi_i)^2}\cos\phi_i$$

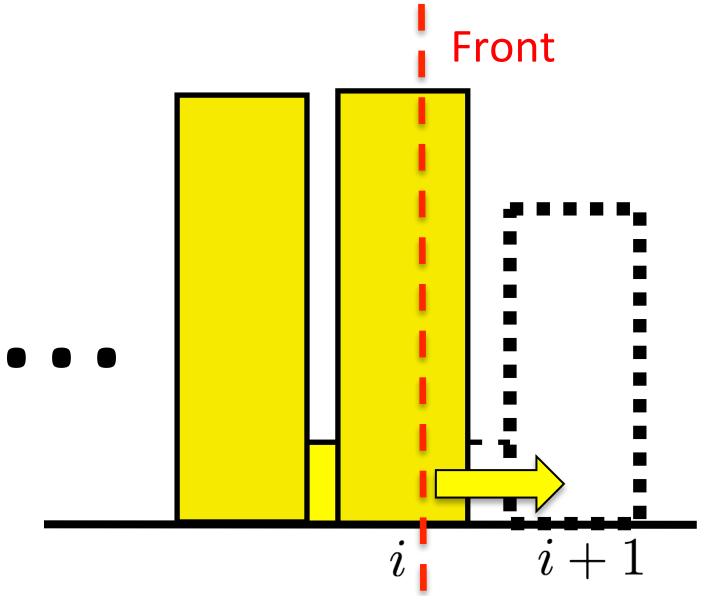
Oscillation model



2. How to spread

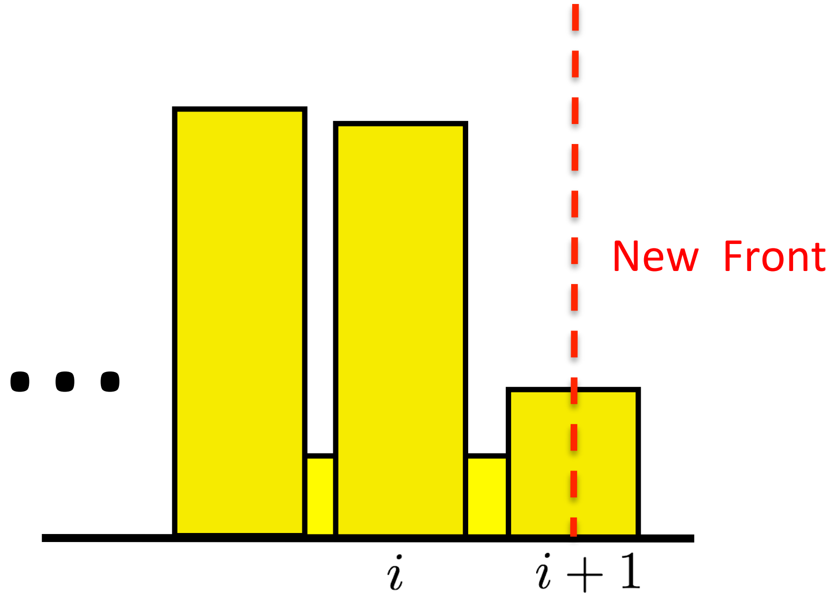


How to move the boundary



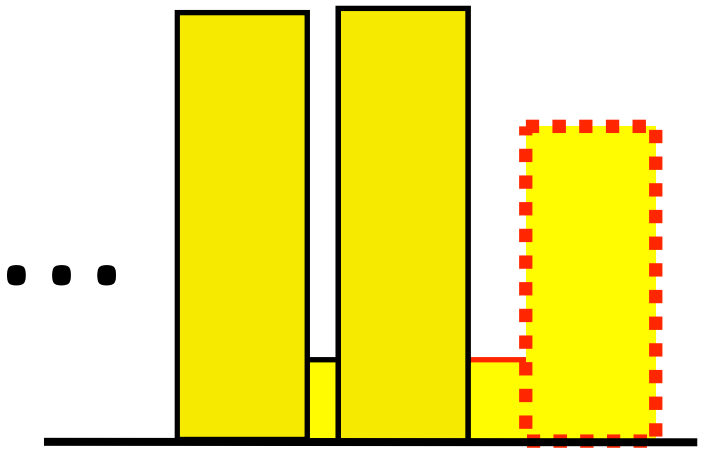
If pressure on boundary exceeds critical value

$$p_i > p_{th}(\eta_i) = \tilde{p}\eta_i$$



New cylinder appears!

Gel degree



soft hard

Gel degree η_i $(\eta_i : 0 \sim 1)$

Condition of breakthrough the edge

$$\Rightarrow \quad p_i > p_{th}(\eta_i) = \tilde{p}\eta_i$$

$$\eta_{i+1} = \eta_{new} \; (= 0.3)$$

Stiffness (strength) of actuator $\kappa_i = \tilde{\kappa}\eta_i$

Stiffness of Membrane $k_{i+\frac{1}{2}} = \tilde{k}(\frac{\eta_i + \eta_{i+1}}{2})$

Conductivity $D_{i+\frac{1}{2}} = \tilde{D}\left(\frac{\eta_i + \eta_{i+1}}{2}\right)^{-1}$

Locomotion model

Oscillation model

$$S\dot{l}_i = D_{i-\frac{1}{2}}(p_{i-1} - p_i) + D_{i+\frac{1}{2}}(p_{i+1} - p_i)$$

$$\dot{\phi}_i = \omega + T_i \Gamma(\phi_i, l_i, \bar{L}_i)$$

$$+$$

Slow dynamics

$$\dot{\eta} = \epsilon_1 (1 - \eta_i)\eta_i$$

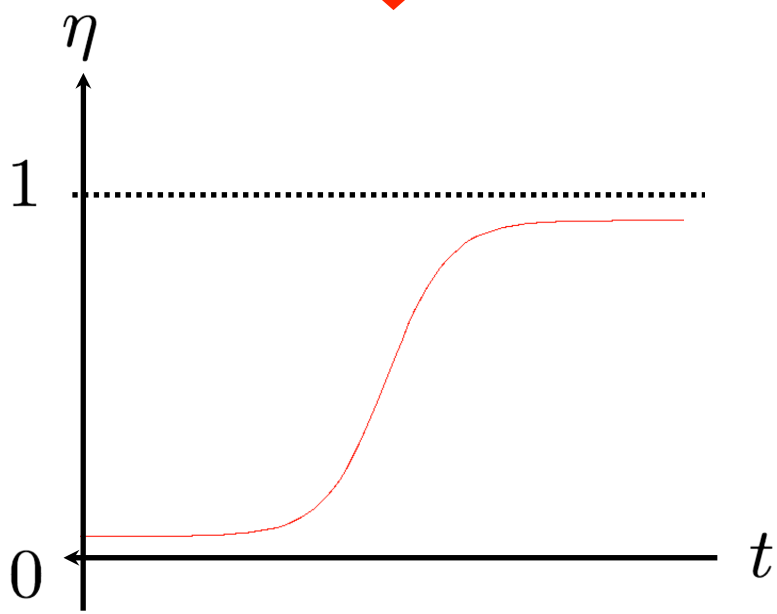
$$\dot{\bar{L}}_i = \epsilon_2 (l_i - \bar{L}_i)$$

$\epsilon_1 = 0.02, \epsilon_2 = 0.01$

Slow dynamics

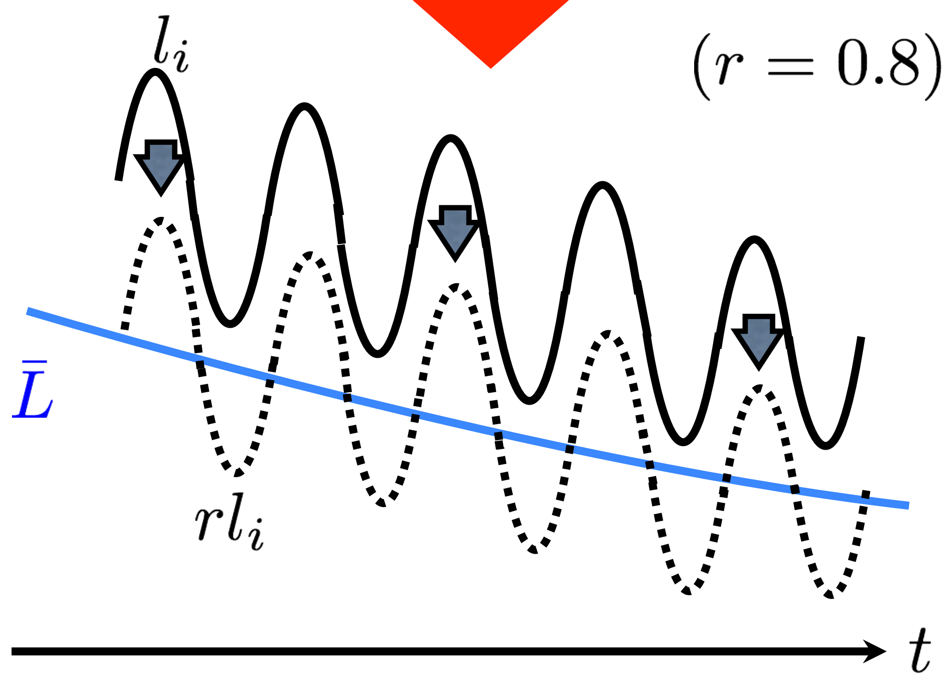
Gel degree

$$\dot{\eta}_i = \alpha(1 - \eta_i)\eta_i$$

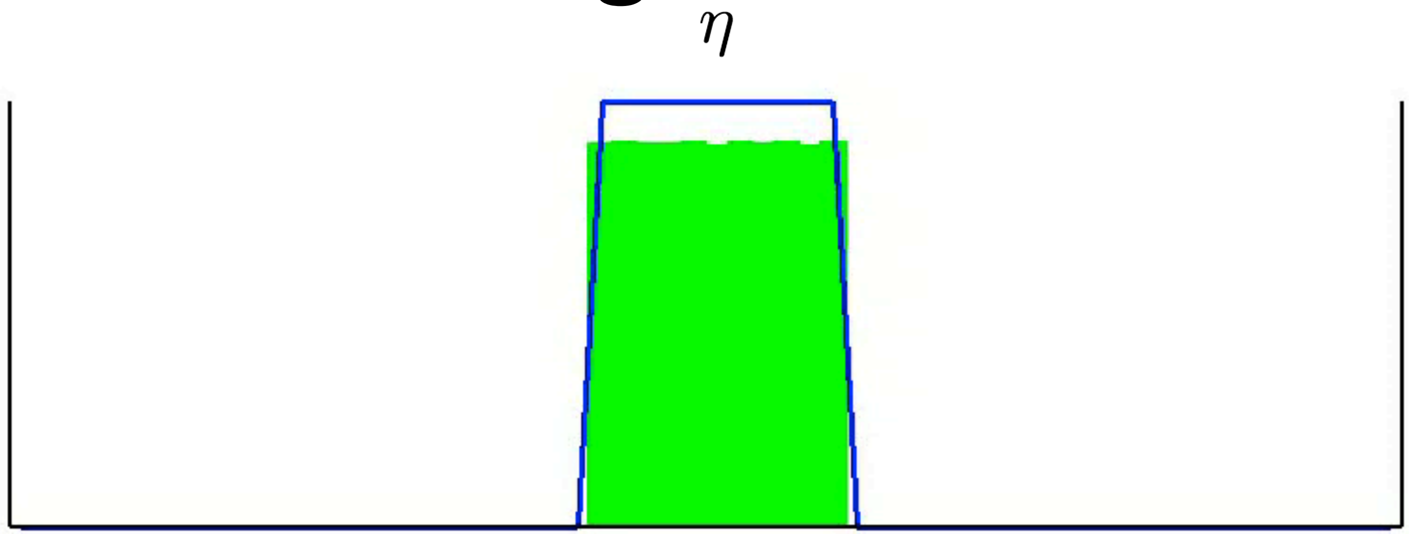


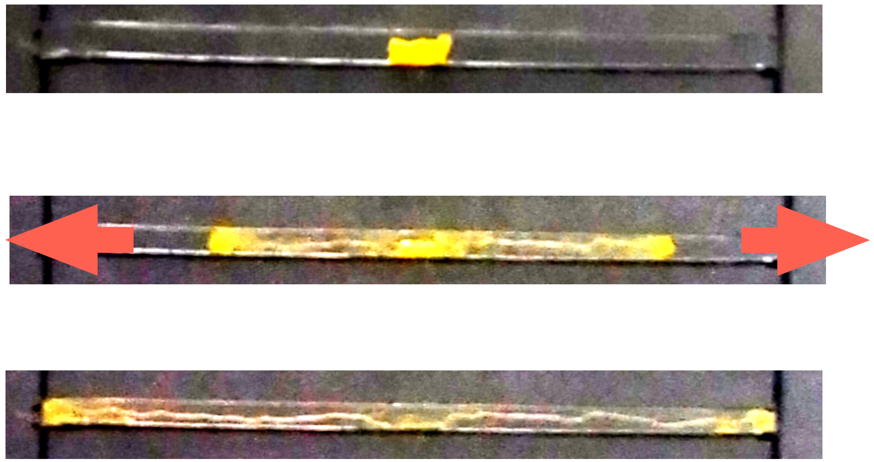
Adaptation of the center of oscillation

$$\dot{\bar{L}}_i = \epsilon(rl_i - \bar{L}_i)$$



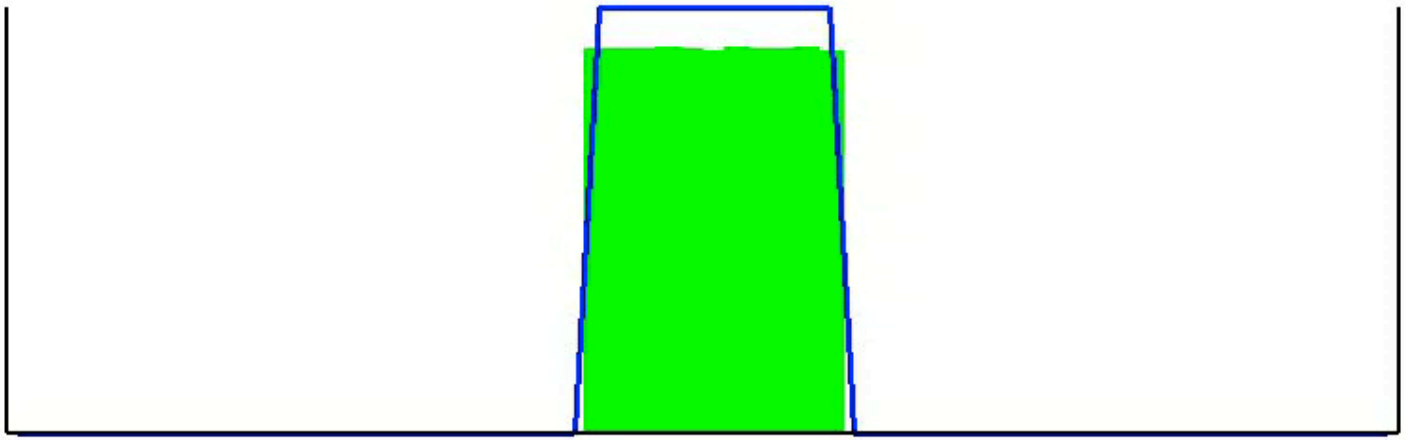
Moving to both sides

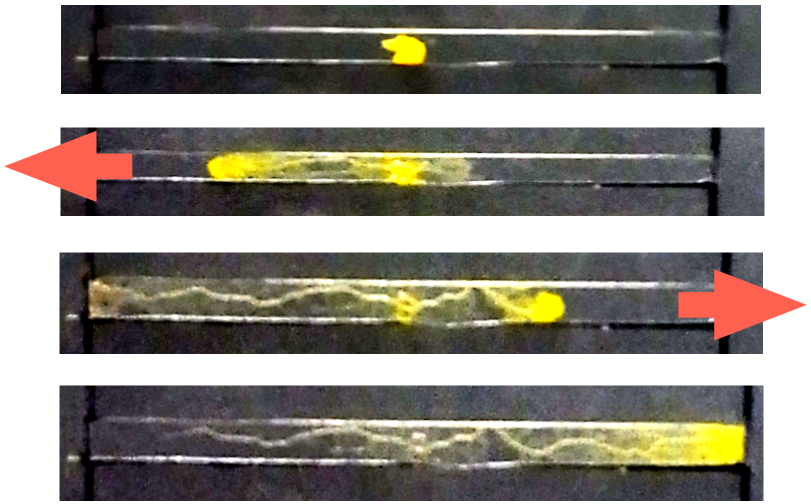




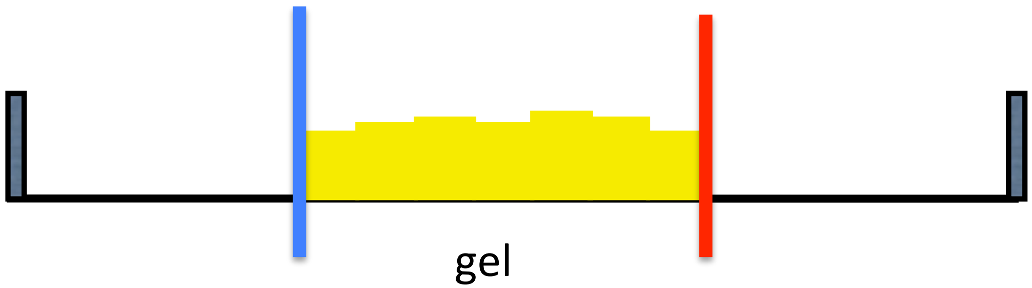
One direction moving

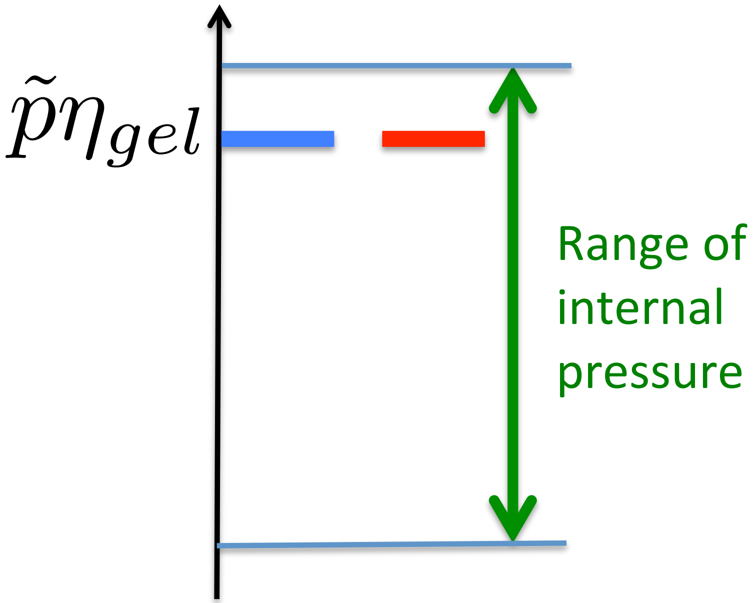
η



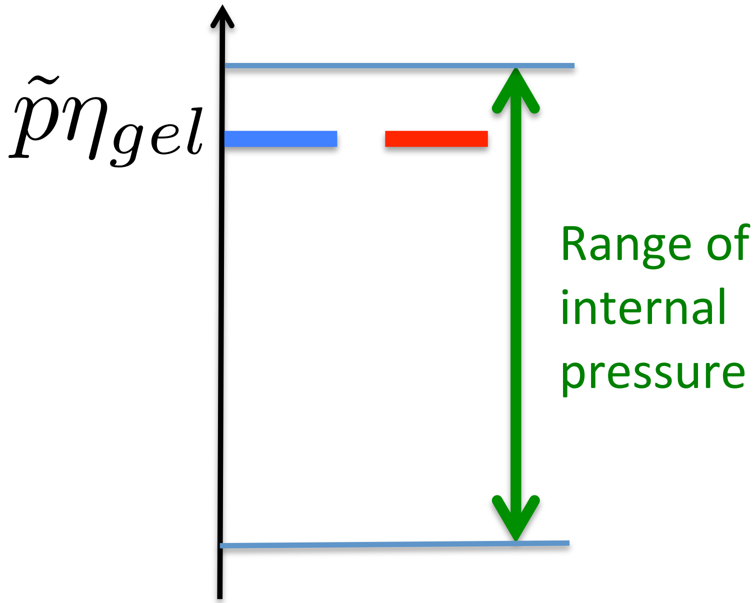


How to move in the opposite direction

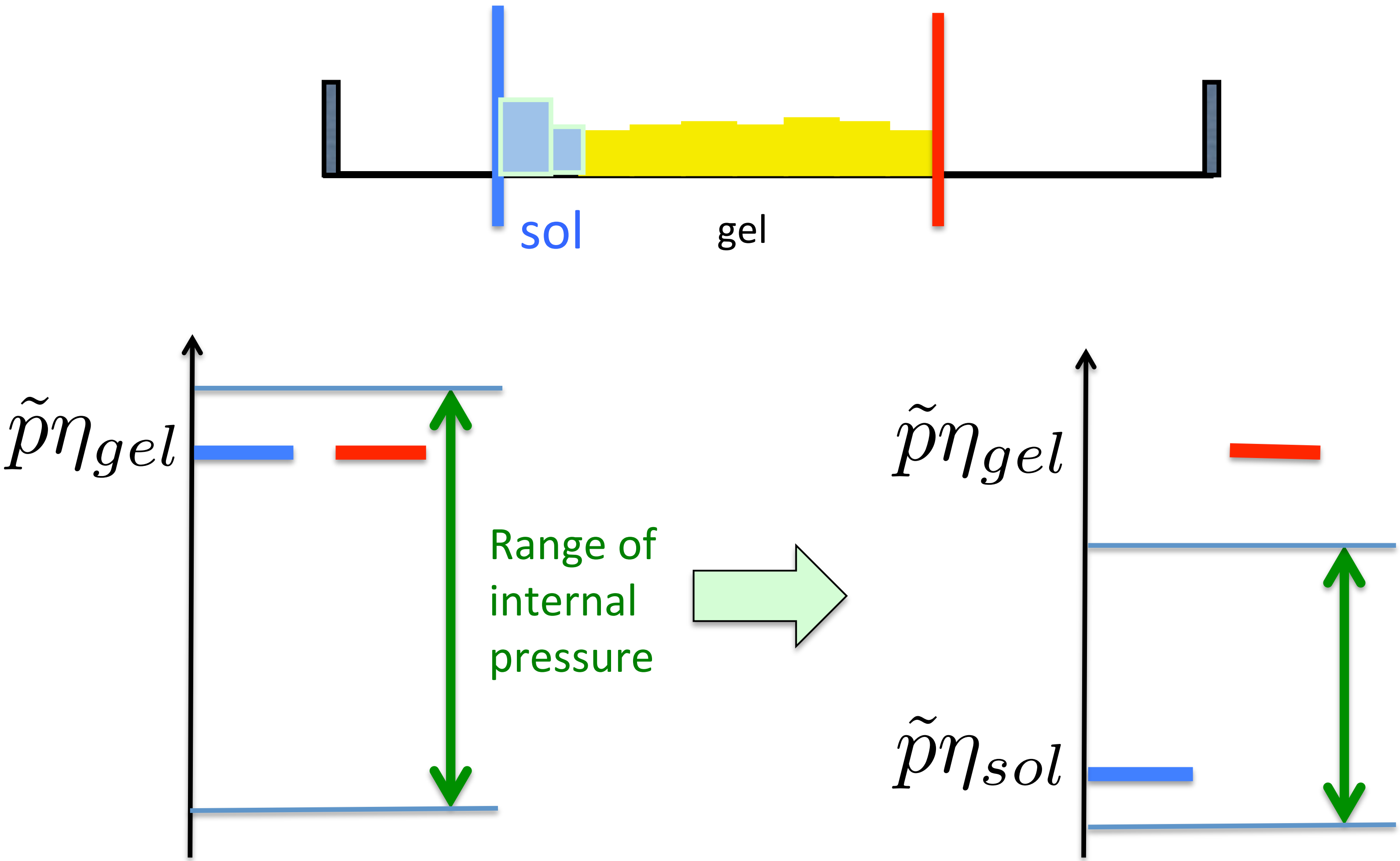




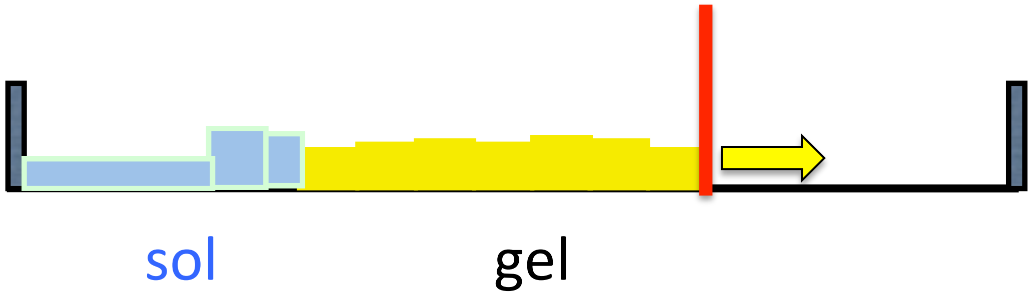
How to move in the opposite direction

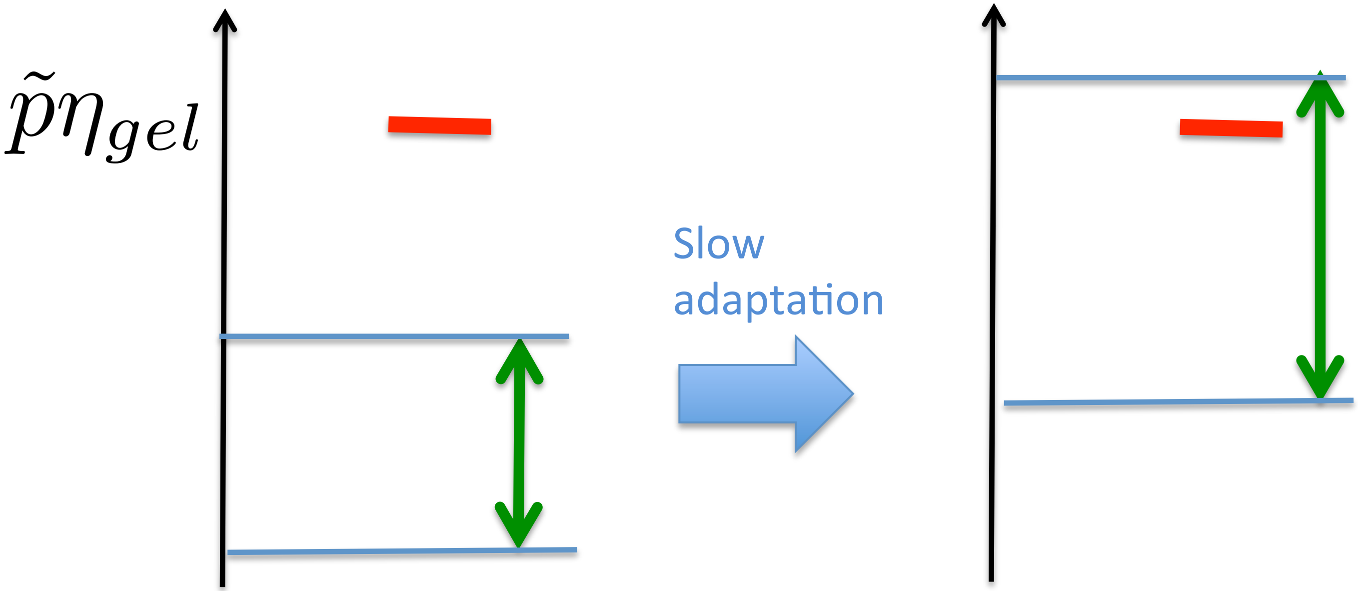


How to move in the opposite direction



After reaching one side





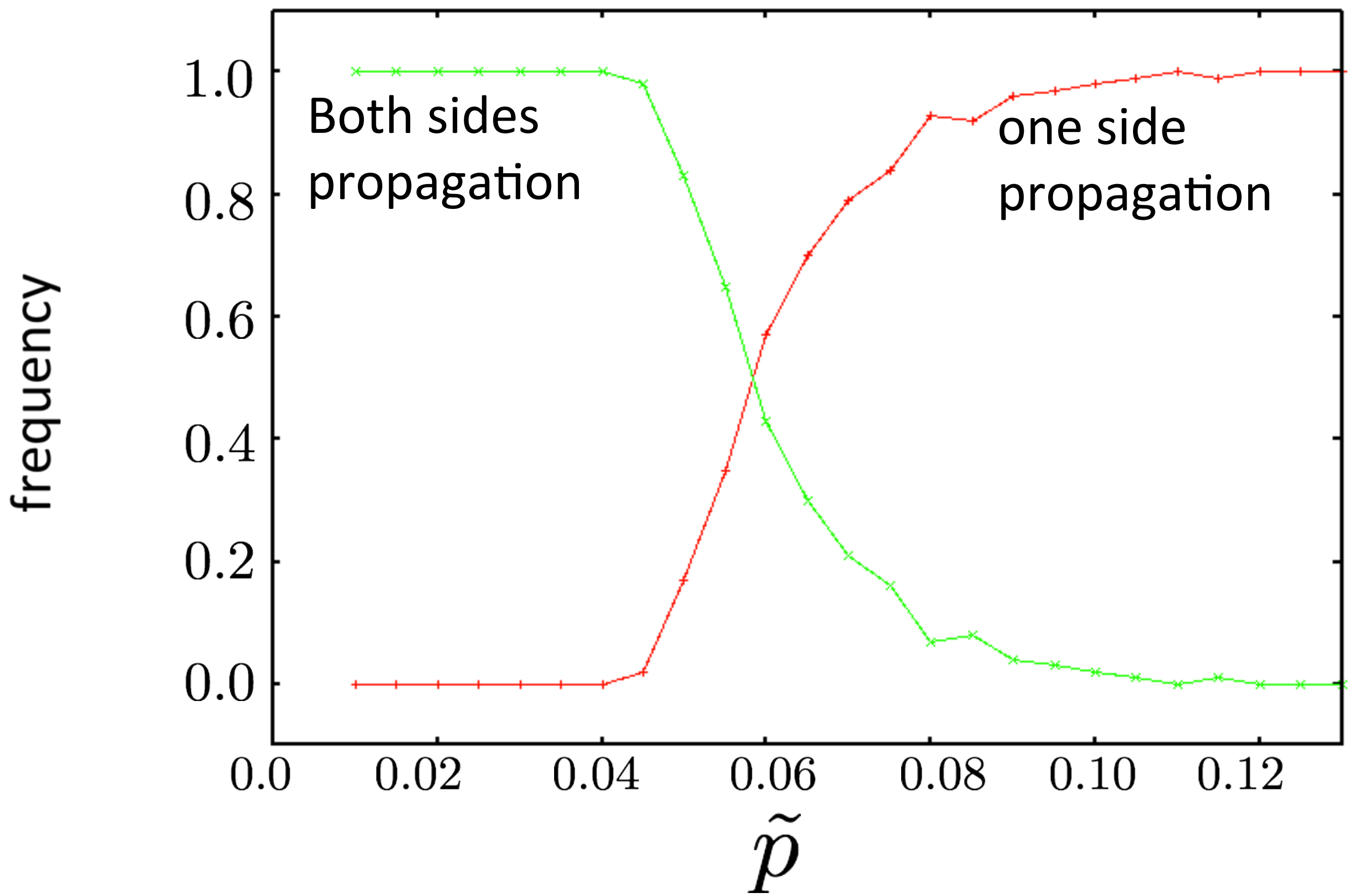
Slow dynamics induce pressure increasing.

Our scenario

- Propagation causes decreasing of internal pressure.

- Slow dynamic, i.e. the transformation from sol to gel and the adaptation of natural length, induces increasing of internal pressure.

Dependency of threshold parameter



Initial condition:

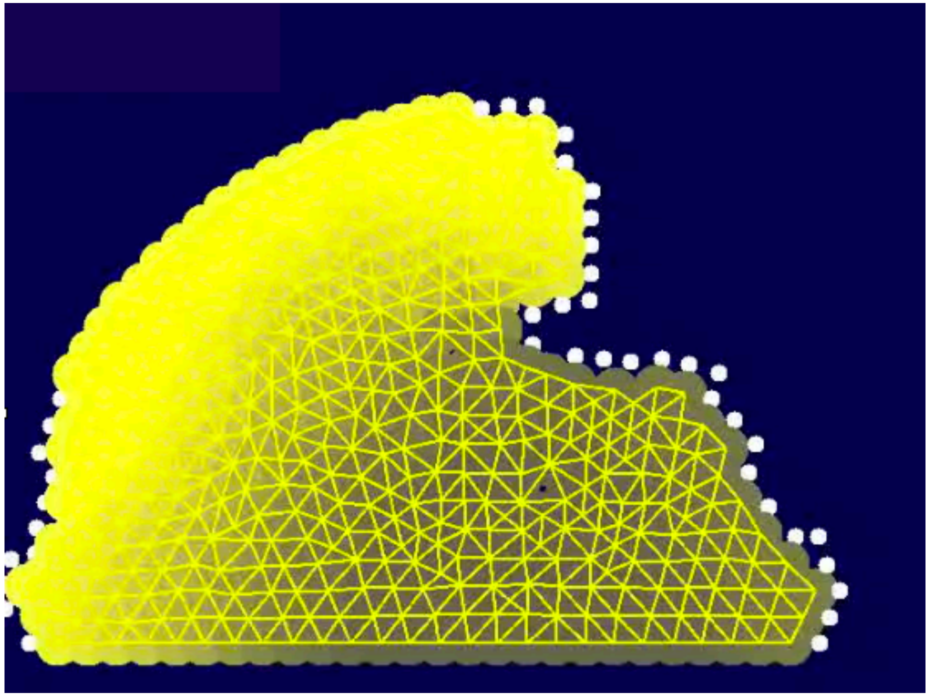
$l_i(0)$: Uniform

$\theta_i(0)$: Random

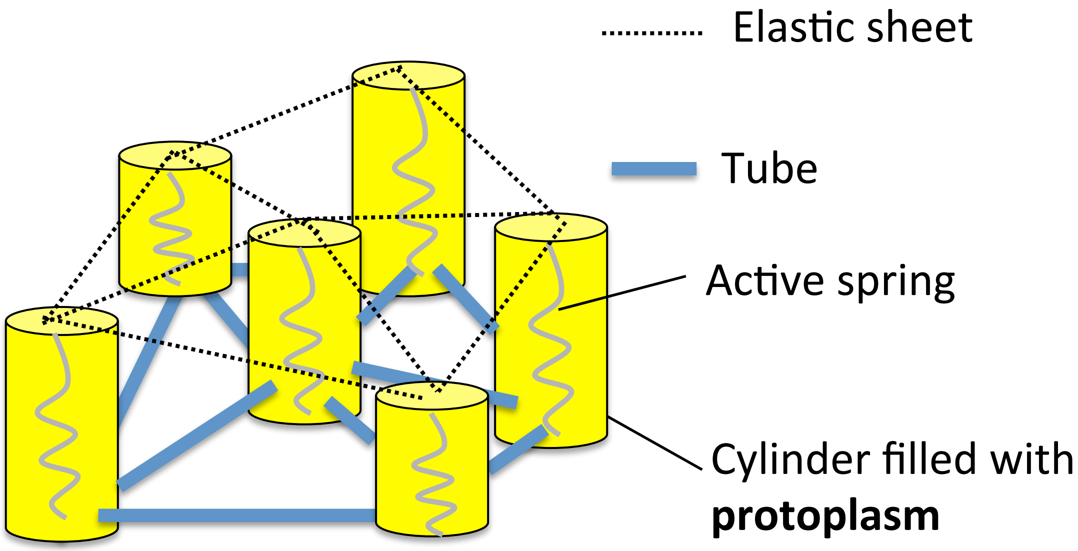
Condition for expand

$$p_i > p_{th}(\eta_i) = \tilde{p}\eta_i$$

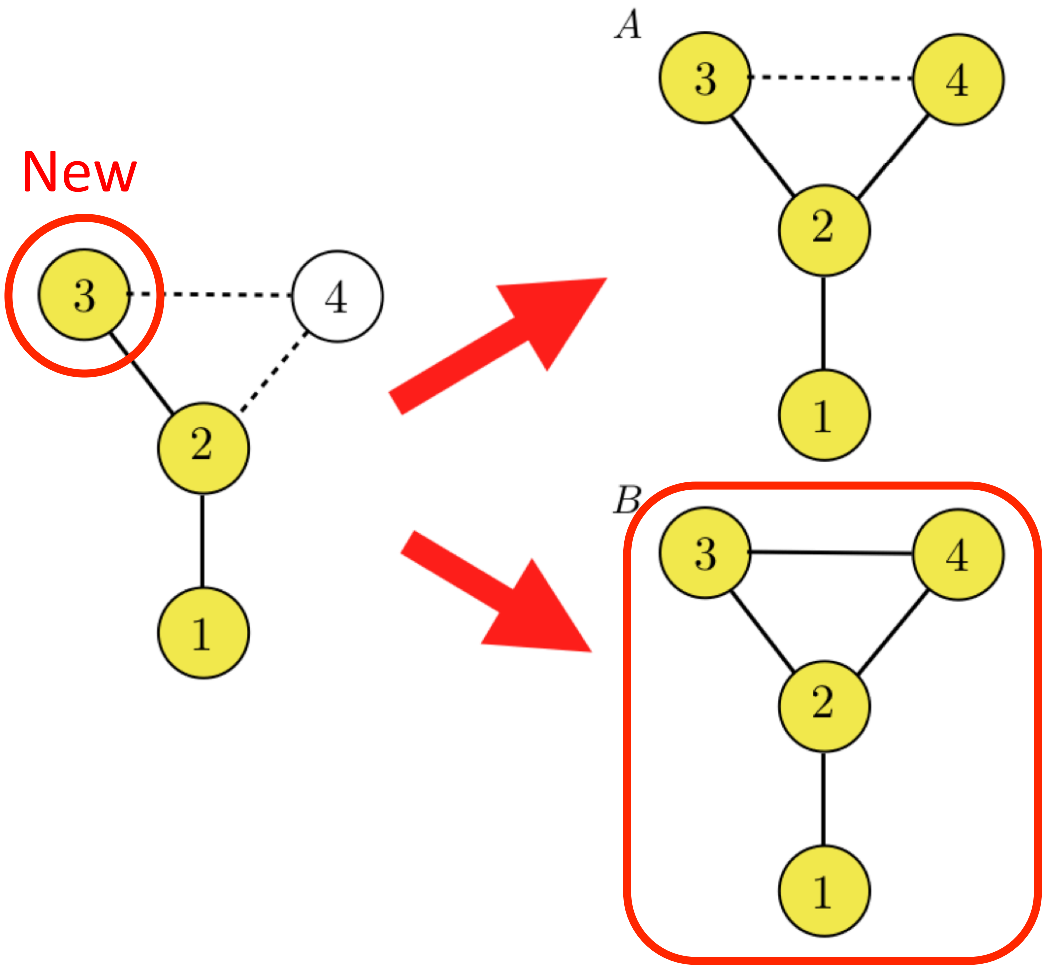
2D simulation

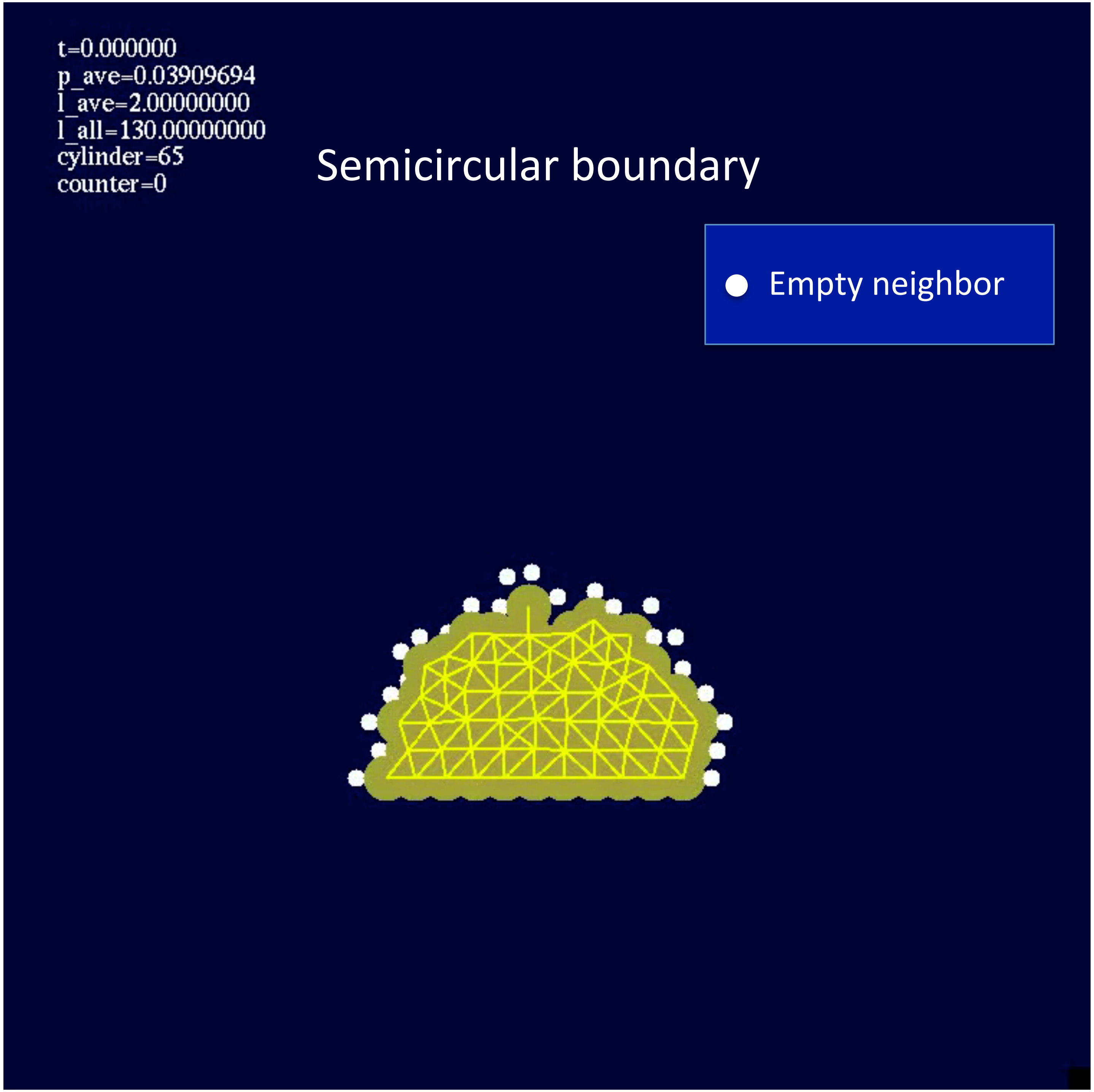


2D model



Elastic sheet
Tube
Active spring
Cylinder filled with
protoplasm


New
3
4
2
1
A
3
4
2
1
B
3
4
2
1


t=0.000000
p_ave=0.03909694
l_ave=2.00000000
l_all=130.00000000
cylinder=65
counter=0
Semicircular boundary
Empty neighbor

Conclusion

- We modeled pressure driven mathematical of physarum locomotion.
- Mechanism of the two types of propagation pattern can be explained with our model.
- Pressure