"Spatial propagation for nonlocal non-autonomous Fisher-KPP equation"

Arnaud Ducrot (Université Le Havre Normandie, France)

In this lecture we discuss existence results of travelling wave solutions and spreading speed for a non-autonomous Fisher-KPP equation with nonlocal diffusion. We prove that under suitable time averaging properties for the coefficients, the equation exhibits a definite spreading speed. We also study non-autonomous Fisher-KPP equation on a lattice and deduce from our analysis—some spreading phenomenon for some predator-prey systems on lattice.

"Ecocultural range-expansion model of modern humans in Paleolithic"

Joe Yuichiro Wakano (Meiji University, Japan)

Modern human range expansion and the resulting extinction or assimilation of archaic humans such as Neanderthals took place roughly 50,000 years ago. This phenomenon is recently very actively studied by using genetic methods such as ancient DNA analysis. In this talk, a reaction diffusion model is proposed with emphasis on archaeological and ecological aspects. Range expansion dynamics are studied as the traveling wave solutions in the system.

"Traveling wave solutions for a three-species diffusive competition system"

Jong-Shenq Guo (Tamkang University, Taiwan)

We shall discuss the existence and stability of traveling waves for a three-species diffusive competition system.

This talk is based on some recent joint works with Karen Guo and Masahiko Shimojo.

"Stability of single transition layer solutions in mass-conserving reaction-diffusion systems with bistable nonlinearity "

Hideo Ikeda (University of Toyama, Japan)

Mass-conserving reaction-diffusion systems with bistable nonlinearity are considered under general assumptions, which are useful models for studying cell polarity formation, whose process is key in cell division and differentiation. The existence of stationary solutions with a single internal transition layer is shown by using the analytical singular perturbation theory. Moreover, a stability criterion for the stationary solutions is provided by calculating the Evans function. This is a joint work with Masataka Kuwamura of Kobe University.

"Traveling wave solutions of combustion in a narrow channel"

Hirofumi Izuhara (Miyazaki University, Japan)

It is reported that combustion in a narrow channel shows a variety of char patterns depending on the airflow rate. In this talk, we consider a mathematical model which describes the combustion experiment, and numerically study the existence of traveling wave solution in one space dimension. In addition, we investigate the instability of its planar combustion wave which is the onset of the variety of char patterns.

"Two phase Stefan problems as the singular limit of competition-diffusion systems arising in population dynamics"

Danielle Hilhorst (Université Paris-Saclay, France)

Competition-diffusion systems are coupled systems of nonlinear parabolic equations, where the unknown functions represent the densities of interacting biological populations. We will first study the singular limit of of a two-component competition-diffusion system in population dynamics when the interspecific competition rate tends to infinity [7], [8]. Using energy estimates, we will prove that the solution converges to the weak solution of a problem with a free boundary, which Mayan Mimura used to call a Stefan problem with zero latent heat [1], [2], [3]. In biological terms, this amounts to proving that the habitats of two interacting populations become completely disjoint in the fast reaction limit. We will then consider a three component competition-diffusion system and prove that its solution converges to a Stefan problem with positive latent heat [4], [6].

Another question involves the limit of the Stefan problem as the latent heat coefficient tends to zero; we will show that it converges to the Stefan problem with zero latent heat [5]. A question which we have started to study is then the following: can we prove a similar result in the case that the partial differential equations in the Stefan problems are perturbed by a white noise in time [9]

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"The floodgates to pattern formation problems"

Yasumasa Nishiura (Hokkaido University, Japan)

W. T. Gowers, a Fields medalist from Cambridge in 1998, eloquently expounded on the pivotal role of Paul Erdős in his essay titled "The Two Cultures of Mathematics." He (Erdős) is famous not because it has large numbers of applications, nor because it is difficult, nor because it solved a long-standing open problem. Its fame rests on the fact that it opened the floodgates to probabilistic arguments in combinatorics.

In a similar vein, Mayan Mimura opened a parallel set of floodgates, ones that lead to the modeling and analysis of reaction-diffusion equations. I am eager to trace the initial footsteps of Mayan and delve into the lasting impact of his contributions on present-day research.

"Keller-Segel type approximation for nonlocal Fokker-Planck equations in one-dimensional bounded domain"

Yoshitaro Tanaka (Future University Hakodate, Japan)

To describe biological phenomena such as cell migration and cell adhesion many evolutional equations are proposed in which an advective convolution term with a suitable integral kernel is imposed. It is well known that such nonlocal equations can reproduce various behaviors depending on the shape of the integral kernel. These nonlocal evolutional equations are often difficult to analyze, and the analytical method is developing. In the light of these background we approximate the nonlocal Fokker-Planck equations by the combination of a Keller-Segel system which is a typical and locally dynamical system. We will show that the solution of the nonlocal Fokker-Planck equation with any even continuous integral kernel can be approximated as a singular limit of the Keller-Segel system with specified parameters.

"Spreading fronts arising from the singular limit of reaction-diffusion systems"

Chang-Hong Wu (National Yang Ming Chiao Tung University, Taiwan)

To gain insight into the formation of spreading fronts of invasive species, in this presentation we will focus on the singular limit of reaction-diffusion systems.

We investigate the dynamics of the limiting systems and give some interpretations for spreading fronts from the modeling viewpoint. The talk is based on joint works with Hirofumi Izuhara and Harunori Monobe.

"Segregation pattern in a reaction-diffusion model of asymmetric cell division"

Yoshihisa Morita (Ryukoku University, Japan)

We deal with a mathematical model describing polarity in the asymmetric cell division of C. elegans embryo. In the maintenance phase of asymmetric cell division anterior PAR protein (aPAR) and posterior PAR protein (pPAR) are exclusively formed and a segregation pattern is created for the polarizations of aPAR and pPAR. Seirin-Lee and Shibata (2015) proposed a 4-component reaction-diffusion system with mass conservation as a model to describe the segregation pattern. Later, some gradient-like dynamics and variational structure in a slightly modied model system were revealed by Morita and Serin-Lee (2021). In this talk we review their work and report a recent progress.

"Speed-up of traveling waves by negative chemotaxis"

Quentin Griette (Université Le Havre Normandie, France)

We consider the traveling wave speed for Fisher-KPP (FKPP) fronts under the influence of chemotaxis and provide an almost complete picture of its asymptotic dependence on parameters representing the strength and length-scale of chemotaxis.

Our study is based on the convergence to the porous medium FKPP traveling wave and a hyperbolic FKPP-Keller-Segel traveling wave in certain asymptotic regimes. In this way, it clarifies the relationship between three equations that have each garnered intense interest on their own. Our proofs involve a variety of techniques ranging from entropy methods and decay of oscillations estimates to a general description of the qualitative behavior to the hyperbolic FKPP-Keller-Segel equation. For this latter equation, we, as a part of our limiting arguments, establish an explicit lower bound on the minimal traveling wave speed and provide a new construction of traveling waves that extends the known existence range to all parameter values.

This is a joint work with Chris Henderson and Olga Turanova.

"A Billiard Problem in Nonlinear Dissipative Systems"

Shin-Ichiro Ei (Hokkaido University, Japan)

The motion of camphor discs in a square domain is considered. Different motions from a usual Billiard problem are observed such as the existence of a stable limit cycle. This talk is mainly done according to the content of the monograph by Miyaji, E. and Mimura. The interaction of elliptic camphor discs is also mentioned.

"Convergence, concentration and critical mass phenomena for a model of cell migration with signal production on the boundary"

Philippe Souplet (Université Sorbonne Paris Nord, France) (joint work with Nicolas MEUNIER (Université d'Evry-Val d'Essonne))

We consider a model of cell migration with signal production on the boundary. It consists in a diffusion equation with nonlinear nonlocal advection, complemented by a no-flux condition ensuring mass conservation.

For nonlinearities with polynomial growth, we first develop a local existence-uniqueness theory in optimal L^p spaces. With help of this tool, we next obtain the following results on the global behavior of solution:

- For small initial data, we have exponential convergence towards a constant.
- If, and only if, the growth of the nonlinearity is at least quadratic, we have concentration, i.e. finite time blowup, for large initial data.
- In the critical case of a quadratic nonlinearity, we observe a critical mass phenomenon in any space dimension (denoting by M the mass of the L^1 initial data):
 - for $M \leq 1$, the solution is global and bounded;
 - for M > 1, there exist initial data leading to finite time concentration.

This critical mass phenomenon is reminiscent of the well-known situation for the 2D Keller-Segel system. The global existence proof is delicate, based on a control of the solution by means of an entropy functional, via an ε -regularity type result.

• Finally we give some partial results on the localization and final profile of the boundary concentration and on the blowup rate.

"Front propagation in the presence of obstacles"

Hiroshi Matano (Meiji University, Japan)

In this talk I will discuss the effect of geometric obstacles on the propagation of fronts. Two types of fronts are considered. One is a transition layer in a bistable reaction diffusion equation. The other is a curvature-dependent motion of plane curves. Both types of fronts are closely related. In the first part, I will present my joint work with Henri Berestycki and François Hamel. I will then discuss the curvature-dependent motion of plane curves through an infinite channel with saw-toothed boundaries. For this second topic, I will first recall my joint work with Ken-Ichi Nakamura and Bendong Lou (2006, 2013), which deal with domains with mildly undulating boundaries. I will then discuss my ongoing joint work with Ryunosuke Mori, which deals with domains whose boundaries have steeper bumps. In such domains, a new type of phenomenon, which we call "obstacle-induced propagation", can be observed.