"Two phase Stefan problems as the singular limit of competition-diffusion systems arising in population dynamics"

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Competition-diffusion systems are coupled systems of nonlinear parabolic equations, where the unknown functions represent the densities of interacting biological populations. We will first study the singular limit of of a two-component competition-diffusion system in population dynamics when the interspecific competition rate tends to infinity [7], [8]. Using energy estimates, we will prove that the solution converges to the weak solution of a problem with a free boundary, which Mayan Mimura used to call a Stefan problem with zero latent heat [1], [2], [3]. In biological terms, this amounts to proving that the habitats of two interacting populations become completely disjoint in the fast reaction limit. We will then consider a three component competition-diffusion system and prove that its solution converges to a Stefan problem with positive latent heat [4], [6].

Another question involves the limit of the Stefan problem as the latent heat coefficient tends to zero; we will show that it converges to the Stefan problem with zero latent heat [5]. A question which we have started to study is then the following : can we prove a similar result in the case that the partial differential equations in the Stefan problems are perturbed by a white noise in time [9]

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