# Multiscale Modeling of Pedestrian Dynamics

## Individuality vs. Collectivity


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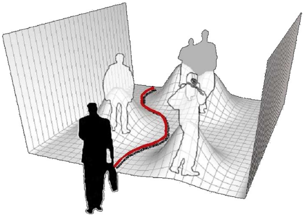
# Moving in a Crowd: Human Perception as a Multiscale Process



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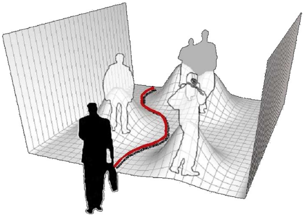

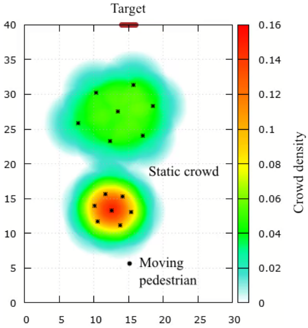

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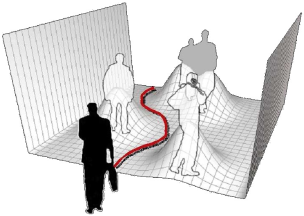

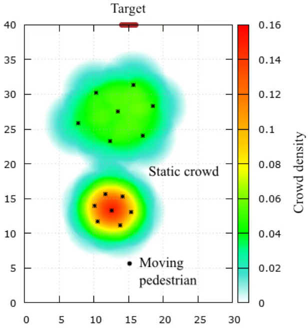


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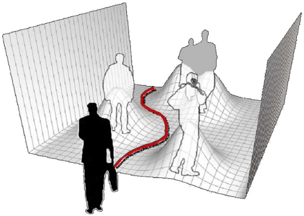

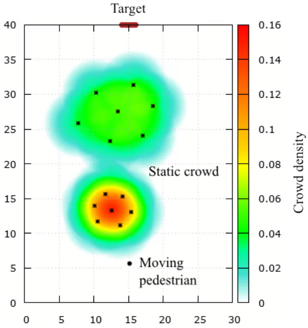


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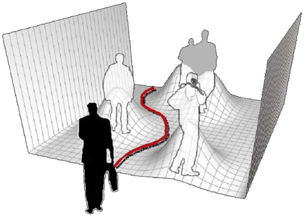

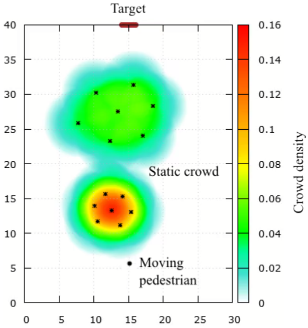


- **Discrete** perception: the moving pedestrian senses the individuals
- **Continuous** perception: the moving pedestrian senses the group
- **Multiscale**: the moving pedestrian tunes his/her perception

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$$\mu = \mathcal{C}_\theta \left( \theta \sum_{k=1}^{N} \delta_{\xi_k} + (1-\theta)\rho \right)$$

  - density $\rho : \mathbb{R}^2 \to [0, +\infty)$ s.t. $\int_{\mathbb{R}^2} \rho(x)\, dx = N$
  - $\theta = \theta(X_t, y) \in [0, 1]$ level of perception
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- $S_R(X_t) \subset \mathbb{R}^2$ sensory region of the moving pedestrian Figure
- $K : \mathbb{R}^2 \to \mathbb{R}^2$ interaction kernel (collision avoidance) Figure

## The Path-Perception Relationship

Assumptions:

- $\theta = \theta(X_t)$, i.e., the level of perception depends on the position of the moving pedestrian ("Lagrangian" perception);
- $\theta : \mathbb{R}^2 \to [0,\, 1]$ Lipschitz continuous;
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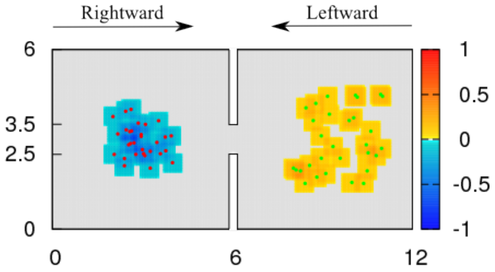
## Theorem (Stability of the trajectories)

*Let $\theta^1$, $\theta^2$ be two perception functions which generate the trajectories $t \mapsto X_t^1$, $t \mapsto X_t^2$, respectively. Fix any final time $0 < T < +\infty$. There exists a constant $\mathcal{C}_T > 0$ such that*

$$\left|X_t^2 - X_t^1\right| \leq \mathcal{C}_T t \cdot e^{\mathcal{C}_T \ell(\theta^1,\, \theta^2) W_1(\rho,\, \epsilon) t} W_1(\rho,\, \epsilon) \left\|\theta^2 - \theta^1\right\|_\infty \qquad \forall\, t \in [0,\, T],$$

*where $\ell(\theta^1,\, \theta^2) := \min\{\mathrm{Lip}(\theta^1),\, \mathrm{Lip}(\theta^2)\}$.*

# Multiscale Modeling of the Whole Crowd Dynamics



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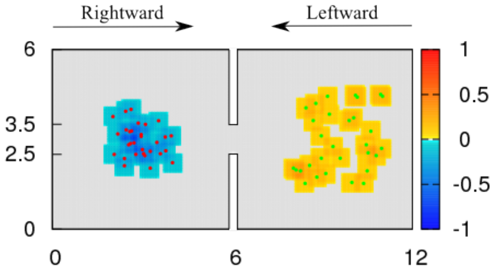


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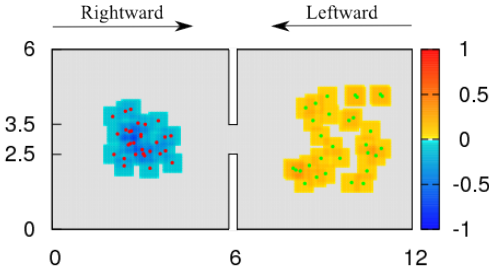


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- **Continuous** perception: clogging of the bottleneck

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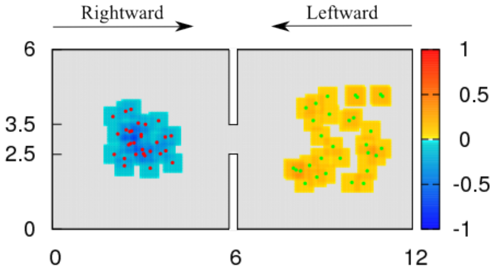


- **Discrete** perception: formation of parallel lanes
- **Continuous** perception: clogging of the bottleneck
- **Multiscale** perception: alternate passage (traffic light effect)

# Modeling by Transport of Measures

- $X_t(x) \in \mathbb{R}^2$ position at time $t$ of the individual who was initially in $x \in \mathbb{R}^2$

$$\dot{X}_t = v[\mu_t](X_t) := v_{\mathrm{d}}(X_t) + \frac{1}{a + \mu_t(S_R(X_t))} \int_{S_R(X_t)} K(y - X_t)\, d\mu_t(y)$$

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- $\mu_t$ as a material quantity: $\mu_0$ is transported to $\mu_t$ by $(t,\, x) \mapsto X_t(x)$

$$\mu_t = X_t \# \mu_0 \;\Leftrightarrow\; \mu_t(E) = \mu_0(X_t^{-1}(E)) \;\Leftrightarrow\; \int_{\mathbb{R}^2} \varphi\, d\mu_t = \int_{\mathbb{R}^2} (\varphi \circ X_t)\, d\mu_0$$

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- Finally

$$\boxed{\partial_t \mu_t + \mathrm{div}(\mu_t v[\mu_t]) = 0} \qquad \text{with } \mu_0 \text{ as initial condition} \qquad (2)$$

# Sketch of the Theory for the Measure-Valued Equation

Assumptions:

- constant $\theta \in [0, 1]$;
- smooth desired velocity, i.e., $v_{\mathrm{d}}$ is Lipschitz continuous in $\mathbb{R}^2$;
- smooth interactions, i.e., $K(\cdot - x)$ is Lipschitz continuous in $S_R(x)$;
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Theorem (Well-posedness)

*Fix a final time $0 < T < +\infty$. There exists a unique weak solution $\mu \in C([0,\, T];\, \mathcal{M}_1^N(\mathbb{R}^2))$ to the Cauchy problem* (2)*, which also satisfies*

$$W_1(\mu_t^1,\, \mu_t^2) \leq \mathcal{C} W_1(\mu_0^1,\, \mu_0^2) \quad \forall\, t \in (0,\, T].$$

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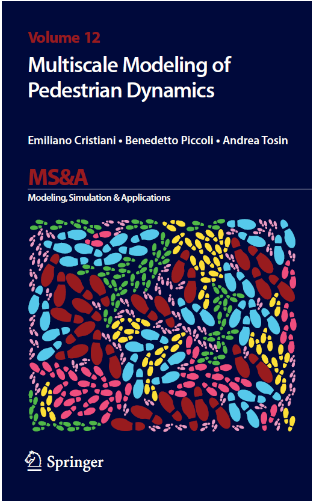
## Theorem (Multiscale representation)

- *If $\mu_0$ is atomic then so is $\mu_t$ for all $t \in (0,\, T]$.*
- *If $\mu_0$ is absolutely continuous and $\mathrm{Lip}(v) T e^{\mathrm{Lip}(v)T} < 1$ then also $\mu_t$ is absolutely continuous for all $t \in (0,\, T]$.*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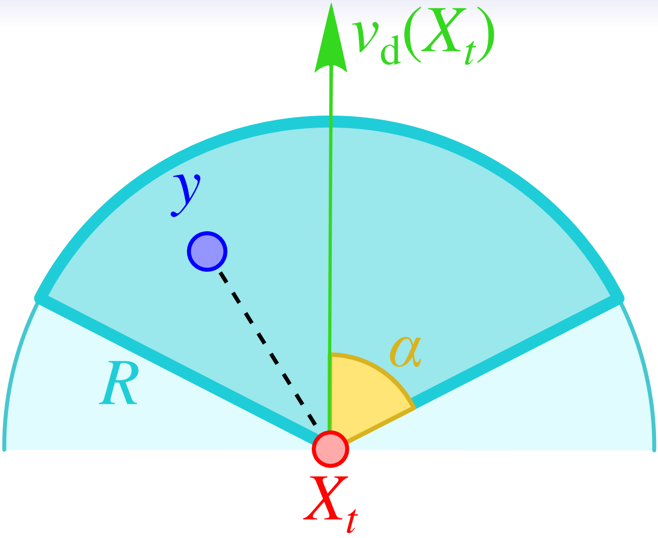


Figure: Sensory region of the moving pedestrian

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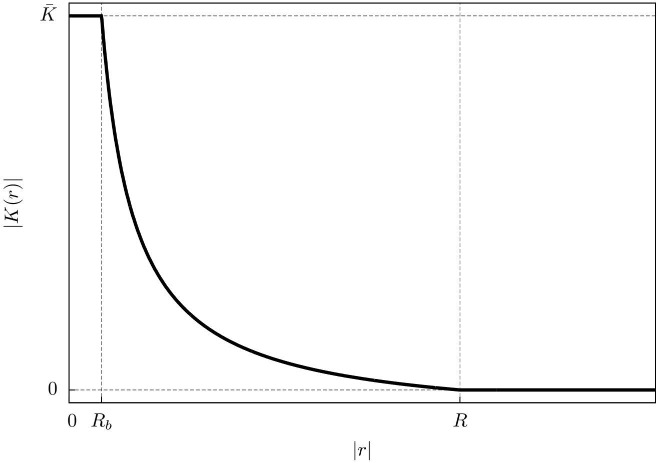


Figure: Modulus of the interaction kernel

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