Jam formation and collective motions of self-driven particles
- Dynamics of dissipative system with asymmetric interaction -

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Self-driven particles as non-equilibrium dissipative many Particle system

Particles with microscopic (local) interaction

Energy in-flow

Energy out-flow (dissipation)

Emergence of macroscopic phenomena

Physical characteristics

I. Gap from Micro to Macro: **Dynamical phase transition**, Bifurcation

II. Emergence of macroscopic **spatial scale**: Pattern formation

III. Emergence of macroscopic **time scale**: Characteristic time, Rhythm

IV. Fluctuation in macroscopic objects: **Power Law** behaviors
Activity in our Research fields

Conferences • Traffic and Granular Flow ‘95～(10, every 2 years)
  • Pedestrian and Evacuation Dynamics ‘99～(7, ”)
  • Traffic Flow Symposium in Japan ‘94～(20, every year)

Access • Traffic Forum  http://www.trafficforum.org
  • Mathematical Society of Traffic Flow
    http://traffic.phys.cs.is.nagoya-u.ac.jp/~mstf/
Flow Dynamics of Self-driven Particles

- Traffic flow (high way)
- Granular flow (e.g. flow in pipe filled with liquid)
- Pedestrians and Evacuation Dynamics
- **Ants’ trail (chemotaxis)**
  - by H. Nishimori

- **Trail formation**

- **Jam of molecular mortars**
  - Jam of Kinesin running on microtubule
  - by Okada, Nishinari

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- Jam of molecular mortars
  - density
  - kinesin
  - 10pM
  - 100pM
  - 1000pM
- Collective bio motions (e.g. bacteria colony)
- Group formation of organisms (e.g. a school of fish)
- Granular flow (e.g. motion of Barchan Dune)

by: H.J. Herrmann
■ Group intelligence of Biological system:
(Slime molds can solve the optimal path in a maze: Nature 407(2000) Nakagaki, et.al.)

■ Control of group motion
- Spiral motion of a school of fish
  stimulus = (enemy)
- Control of sheep
  stimulus = (dog)
  controlled by a tiny stimulus
1. Jam formation in traffic flow
the first work of collective motion of self-driven particles

- Fundamental Diagram
- Critical Car-Density
- Velocity of Jam Cluster
- Following behavior
- Flow upstream of Bottleneck
Fundamental Diagram (highway traffic)

Relation (car-density – flow rate) at a fixed measurement point

Tomei 172kp ’96 Jan.

Plot of 1-month data

Running lane, Passing lane

Flow rate

Free Flow

Jam Flow

Critical car-density

Car density
Flow v.s. car-density relation for several points

Critical density ~ 25 (1/ km)
Velocity of a jam cluster (space-time plot of car motion)

- Measurement by loop-coil detectors (Jam in upstream of a tunnel) by JTC (1999)

- Experiment of circular track (2003)

- Aerial photograph of "phantom jam" : (U.S. A. 1974)

- 「Jam cluster (in city highway in Tokyo) moves backward against a traffic flow at the same velocity about 20km/h」交通工学通論 (by Koshi, 技術書院 1989)
2. Mathematical model for Dissipative System with **Asymmetric** Interaction (Asymmetric Dissipative System) introduced for a model of traffic flow

Asymmetric Non-linear interacting particles with Dissipative term

\[
\frac{d^2 x_n}{dt^2} = a \left\{ V(\Delta x_n) - \frac{dx_n}{dt} \right\}
\]

the direction of movement

\[ n-1 \rightarrow n \rightarrow n+1 \]

\( x_n \): the position of the \( n \)th car
\( \Delta x_n \): the headway (distance) = \( x_{n+1} - x_n \)

\( a \): sensitivity constant : (1/time or 1/mass), parameter for inertia

\( V(\Delta x) \): OV-function \( \rightarrow \) the safe velocity for the headway

non-linear interaction with a particle in front:

optimal velocity or asymmetric force

e.g.: \( V(\Delta x) = \alpha \{ \tanh(\Delta x - d) + \tanh d \} \), Heaviside step function

Interacting particles controlled to optimize each velocity to \( V(\Delta x) \).
Asymmetric Non-linear interacting particles with Dissipation

- **Momentum non-conservation**
  - Totally Asymmetric (OVM)
    
  \[
  \frac{d^2 x_n}{dt^2} = a \left\{ V(\Delta x_n) - \frac{dx_n}{dt} \right\}
  \]
  
  (completely asymmetric, nonlinear interaction)
  
  e.g. \( V(\Delta x) = \alpha \tanh(\Delta x - d) \)
  
- Asymmetric (forward - backward OVM)
  
  \[
  \frac{d^2 x_n}{dt^2} = a \left\{ V(\Delta x_n) - W(\Delta x_{n-1}) - \frac{dx_n}{dt} \right\}
  \]
  
  e.g. \( W(\Delta x) = (\alpha \rightarrow \beta) \)
  
- **Energy non-conservation**
  
- **Not a Potential Force**

- **symmetric**
  
  \[
  \frac{d^2 x_n}{dt^2} = a \left\{ V(\Delta x_n) - V(\Delta x_{n-1}) - \frac{dx_n}{dt} \right\}
  \]
  
  e.g. Toda lattice
  
  \( V(\Delta x) = 1 - e^{-b\Delta x} \)

- **linear**
  
  \[
  \frac{d^2 x_n}{dt^2} = a \left\{ V'(b)(x_{n+1} - x_n) - V'(b)(x_n - x_{n-1}) - \frac{dx_n}{dt} \right\}
  \]

- **nonlinear oscillator**
  
  \( \epsilon \)

- **Totally Asymmetric** (OVM)
  
  Momentum non-conservation

- **Energy non-conservation**

- **Not a Potential Force**
Asymmetric Interaction generates the instability of a homogeneous flow state

**general OV model**

\[
\frac{d^2 x_n}{dt^2} = a \left\{ V(\Delta x_n) - W(\Delta x_{n-1}) - \frac{dx_n}{dt} \right\}
\]

**Linear analysis:** \( y = \exp(ikn + zt) \), small deviation, \( y \) beyond a homogeneous flow

\[
z \equiv \sigma - i\omega = ikz_1 + (ik)^2z_2 + o(k^3), \quad k \sim \text{small}
\]

Dispersion relation by long wave-length expansion

- **Totally asymmetric (OV model):** \( \sigma > 0 \quad \rightarrow \text{unstable} \)

\[
z(k) = ikV'(b) + k^2 \left\{ \frac{2V'(b)}{a} - 1 \right\} V'(b) + o(k^3)
\]

\( b = L/N: \text{average headway} \) (1/density)  

- **Asymmetric:**

\[
z(k) = ik\{V'(b) - W'(b)\} + k^2 \left\{ \frac{2[V'(b) - W'(b)]^2}{a[V'(b) + W'(b)]} - 1 \right\} \frac{V'(b) + W'(b)}{2} + o(k^3)
\]

- **symmetric int. + dissipation:**

\[
z(k) = -k^2V'(b) + o(k^3)
\]

Critical condition

\( W = 0 \)

\( W = V \)

(absolutely stable)
3. Dynamics of Asymmetric Dissipative System

- Dynamical phase transition
- Bifurcation - stability change of solutions -
- Emergence of moving cluster (macroscopic spatial-scale)
- Induced time (macroscopic temporal-scale)
- Power law behavior
Space-time plot for time evolution of forming a cluster

Homogeneous flow solution: unstable, \( a < 2V'(b) \)

Cluster flow solution: stable

Profile of cluster solution
Dynamical phase transition

Homogeneous flow solution: unstable \( a < 2V' \)

Moving cluster flow solution: stable

Emergence of moving cluster

"Limit Cycle"

Non-equilibrium balance of in-and-out flow of particles

\( \Leftrightarrow \) Particles move as the same way with

Induced time-delay \( \tau \) in changing through a cluster.

Forming a macroscopic object with its own motion:

\[
\dot{v}_c = -\frac{\Delta x_j}{\tau}
\]
4. Experiment, Observational data v.s. Theory of Asymmetric dissipative system
Experiment of Jam formation on Circular track v.s. Simulation of Mathematical model


Simulation in OV model:
- $L=230m, N=22. \ a < 2V'(b)$
- OV-function: $v_{\text{max}}=40\text{km/h}, \ \Delta x>20\text{m}$

Experiment:
- $L=230m, N=22.$

After 5 min. a Jam cluster was formed and stable.
Experiment of Jam formation on Circular track
Traces of all cars on the circular track

~20 km/h

N = 22, L = 230 m

N = 23, L = 230 m
Lecture in Santa Fe Institute

Symmetry Breaking, Phase Transition and Non-Equilibrium Phenomena

→ santa fe institute lectures symmetry

Depression temporarily slowed the planet's warming. The analysis also suggests that the Montreal Protocol, which phased out chemicals that deplete the ozone layer and trap heat, has helped to slow warming in recent decades. Nature Geosci. http://doi.org/p2b (2013)

For a longer story on this research,

Traffic jams follow the laws of physics

Traffic congestion closely resembles the physics of phase transitions, such as when ice melts or a metal becomes superconducting. Shin-ichi Tadaki at Saga University in Japan and his colleagues used a high-resolution laser scanner to track cars travelling around an empty indoor baseball stadium, then analysed those data as if they were studying phase transitions in a material. They found that above a critical density of cars, traffic flow became unstable and changed from free-flowing to a jam. Scaled up, that density value fits with those seen on real-world motorways, the authors say. New J. Phys. 15, 103034 (2013)

Homogeneous flow solution

Jam flow solution

Critical density

\[ a = 2V'(\rho_c) \]

\[ 25 \text{ cars/(1 km)} \]

Flow rate

Car density

Background: observed data

Calculations by theory of OV model

Observational data in highway v.s. OV Theory

\[ q \ (1/5\text{min}) \]

\[ \rho \ (1/\text{Km}) \]

Tomei 172kp '96 Jan.
5. Mathematical aspects of Asymmetric Dissipative System

- Dynamical phase transition - many-particles system -

  Hopf Bifurcation
  - stability change of solutions -

- Emergence of moving cluster
  (macroscopic spatial-scale)
Property of Dynamical phase Transition

Phase Diagram

- a*: sensitivity
- b*: mean headway

- i) homogeneous flow: stable
- ii) homogeneous flow, jam flow: both stable (Bistable phase)
- iii) jam flow: stable

 critical curve of linear stability a=2V′(b)

Hopf bifurcation

- b=b*: trivial solution
- b≠b*: subcritical Hopf bifurcation
Asymmetric Interaction \rightarrow \text{Hopf-bifurcation}

Physical. Review. E 80, 026203 (2009)

\[ \frac{d^2 x_n}{dt^2} = a \left\{ V(\Delta x_n) - W(\Delta x_{n-1}) - \frac{dx_n}{dt} \right\} \]

The linear equation of motion for small deviation, \( y \) beyond a homogeneous flow.

\( y_n = \exp(ink + zt), \quad z \equiv \sigma - i\omega \)

\[
\begin{cases}
\sigma^2 - \omega^2 = a(V'(b) + W'(b)) \cos \theta - a\sigma \\
-2\sigma \omega = a(V'(b) - W'(b)) \sin \theta + a\omega
\end{cases}
\]

\( V = W \rightarrow \omega = 0 \) only, at bifurcation point: \( \sigma = 0 \)

i.e. No bifurcation

\( V \neq W \rightarrow \text{Eq. of } \omega^2 \), which always has positive solution in \( a < 2V'(b) \).

\( \Rightarrow \pm \omega \) are conjugate solutions at \( \sigma = 0 \)

i.e. \text{Hopf-bifurcation}
6. Specific Properties in Asymmetric Dissipative System

- $N$ (# of Particles) - dependence
- Properties of Emerged Objects
Cluster emerges even in $N=3$.
Limit cycle (Jam flow) for $N$ (#of particles) and $L/N$ (density), $a=1$

$N=4$

$N=5$

$N=10$

$N=30$

e.g. $V(\Delta x) = \alpha \{ \tanh(\Delta x - d) + \tanh d \}$

d : the inflection point (1/density)

\[ e^{-Na\tau} \]

Asymmetric Dissipative System

- Limit cycle solutions converges very rapidly to the universal solution, independent on $N$ and $L/N$. 

Universality of Jam
7. Instability in 2-dim. System and Group formation

- 2-dimensional OV model
- Higher dimensional modes
  - longitudinal mode (the same as 1-dim)
  - transverse mode
  - elliptically polarized mode

Granular, discrete particles
2-dimensional OV Model for Biological Motion or Pedestrians

Basic idea:
An organism maintains an optimal velocity which depends on distances to others.

Eq. of motion:
\[
\frac{d^2x_n(t)}{dt^2} = a \left[ \sum_k F(\Delta x_{kn}(t)) - \frac{d}{dt} x_n(t) \right]
\]

\[
F(\Delta x_{kn}) = \begin{cases} 
\frac{1}{2} \left( \frac{1 + \cos \theta}{n_{kn}} \right) \frac{\tanh 4(\Delta x_{kn} - 1) + c}{2} 
\end{cases}
\]

\[
: \Delta x_{kn} < 2
\]

\[
: \Delta x_{kn} \geq 2
\]
Phase diagram and Behaviors of the exclusive model (Pedestrian flow in a corridor) \( C = -1 \)


Uni-directional Flow

longitudinal wave

jam formation

Counter Flow

transverse wave

triangular lattice structure (homogeneous flow)

blocking state

lane separation

\( \rho \)
Group formation is induced by Elliptically polarized mode in $C \neq -1$

$c = -1$

(if a tiny attractive interaction exists!)

Random motion and Variation of Emerged Object of Deterministic Motions

2-dimensional OV model

Rapid Response to stimulus
2d-OV Simulations
Formation of solutions for a simple maze

Initial state (random motions)

Intermediate state (creating strings)

(a=20.0, N=128)

Solutions of optimal path

- Periodic boundary
- Elastic boundary

Solutions of periodic boundary

Solutions of elastic boundary
8. **Coarse Analysis**

Analysis of macroscopic dynamical state in collective motions of microscopic variables

- Diffusion Map
- Kantorovich metric space
- Equation-free method

*E.g.* Motions of 2-d OV

(Search for the solution of a maze)

in Kantorovich space

→ see the poster No.9 by R. Ishiwata (Nagoya Univ.)
## 9. Summary

**Non-equilibrium Dissipative System with Asymmetric Interaction**

- Inseparable connection between **Asymmetry of Interaction and Dissipation** → Dynamical behavior
- **Density** is a control parameter.

**N(# of particles)-dependency**

- Instability and Phase transition emerges in small N. (N ≥ 3 in OVM)
- Small- N is large enough number as many-body system.

**Emerged macroscopic Objects**

- **Similarity** between Temporal and Spatial structures
- **Rapid Response** to stimulus
- **High Degeneracy** in “local symmetry”

**Energy-momentum Conserved System with Symmetric Interaction**

- Dissipation → **Static state**.
- Density can not be a control parameter.

**Phase transition appears in large N. (strictly, in N→∞)**

- **Slow Response**
- Degeneracy in **global symmetry**
ナゴヤドーム渋滞形成実験
渋滞はなぜ発生するのか -その数理と実証実験映像-

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・ M. Fukui(Nakanihon Automotive College)
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