Sovereign Credit Risk Analysis through Statistical Modeling

Yoko Tanokura

Meiji University

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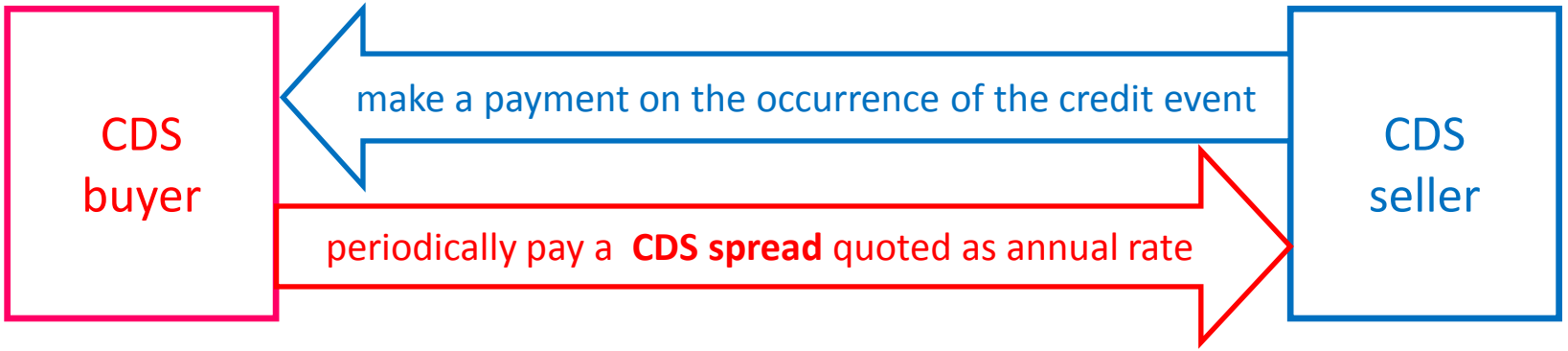
This is the joint work with
Hiroshi Tsuda(Doshisha Univ.), Seisho Sato(Univ. of Tokyo) and Genshiro Kitagawa(ROIS)

Measuring sovereign credit risk

Sovereign Credit Default Swap (SovCDS)

...is an insurance contract that protects the buyer against the issuer's credit risk of the country's debt.



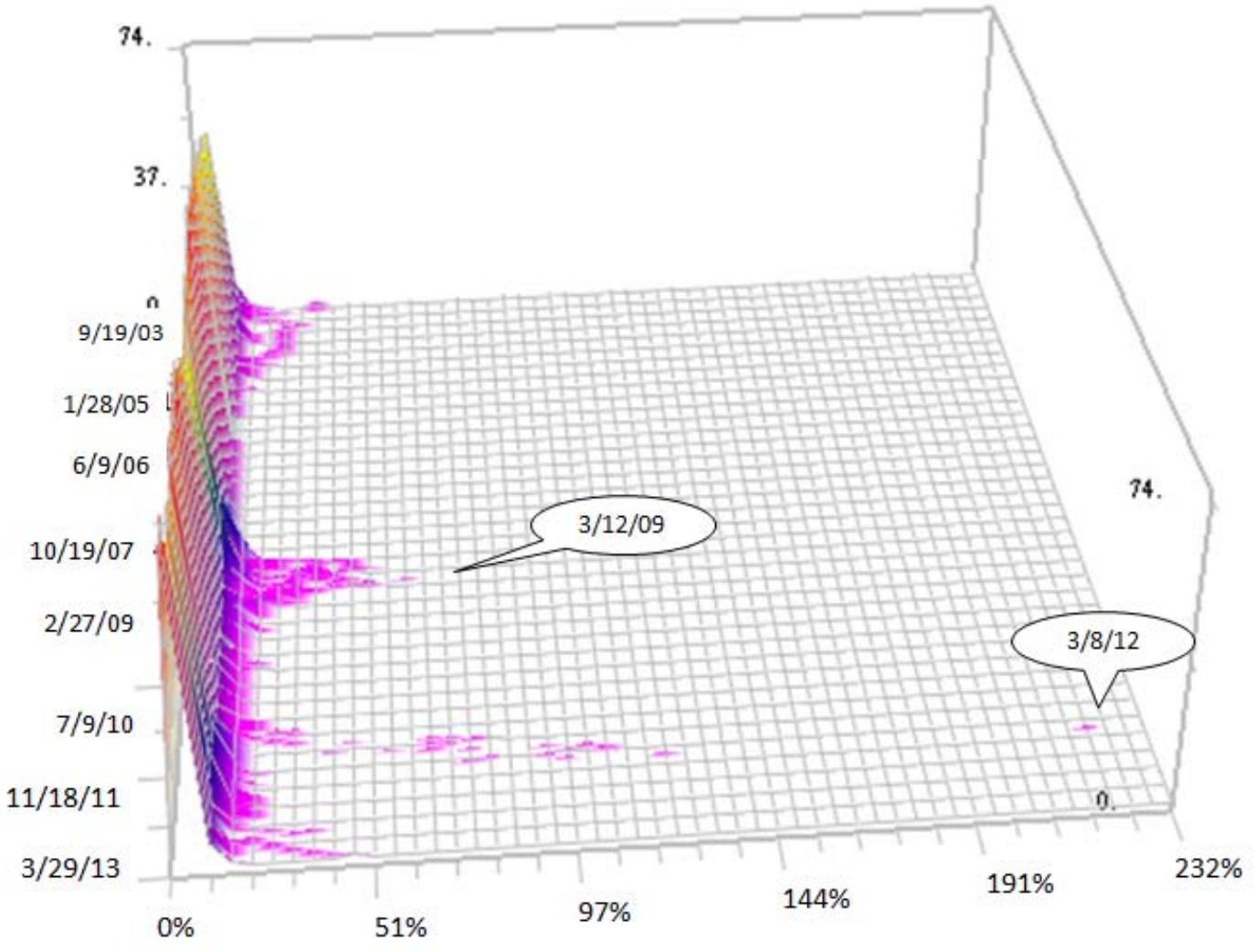
...can be regarded as the market evaluation on the credit risk for the country's economy.

Strictly speaking, although CDS spreads may include other factors such as risk premiums caused by its fluctuations, it is not easy to extract only credit risk.

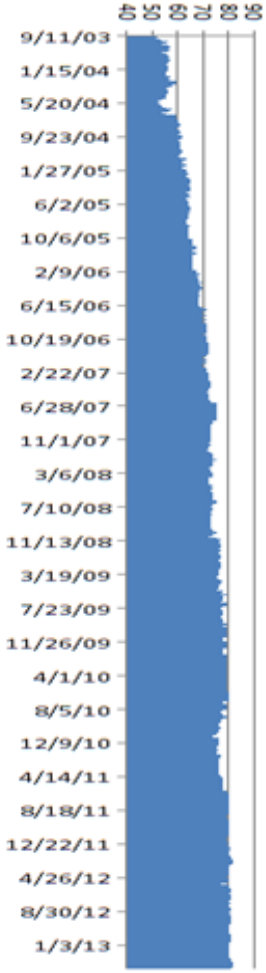
Risk premium is practically assumed to be none or constant.

We suppose that CDS spreads measure the sovereign credit risk of the concerned country's debt.

Time series of SovCDS spread distributions



Number of observations



Source: Markit

Problems:

- Heavy-tailed distributions
- Time-varying number of observations

Method of distribution dependent index construction*

1 Apply the Box-Cox transformation (Box and Cox 1964) to the spreads

$$q_{i,\lambda}(n) = h(p_i(n)) = \lambda^{-1}(p_i(n)^{\lambda} - 1) \quad \lambda \neq 0$$
$$= \log p_i(n) \quad \lambda = 0$$

$n = 1,\ldots,T$: time

$i = 1,\ldots,k(n)$: number of observations

2 For each λ, fit the following trend model to the mean time series $y_\lambda(n)$ of $q_{i,\lambda}(n)$

$$\nabla^l t_\lambda = v_\lambda(n), \qquad v_\lambda(n) \sim N(0, \tau\lambda^2)$$
$$y_\lambda(n) = t_\lambda(n) + w_\lambda(n), \quad w_\lambda(n) \sim N(0,\ \sigma_\lambda(n)^2 / k(n))$$

$$\nabla t_\lambda(n) = t_\lambda(n) - t_\lambda(n-1)$$

$\sigma_\lambda(n)^2$ is estimated by a time-varying variance model (Kitagawa 1987).

3 Estimate parameters by applying state space modeling (Kitagawa 2010)

$$x_\lambda(n) = F\,x_\lambda(n-1) + G v_\lambda(n)$$
$$y_\lambda(n) = H\,x_\lambda(n) + w_\lambda(n)$$

4 Determine an optimal λ by minimizing AIC'_λ : modified AIC_λ (Akaike 1973) to the original spreads (Kitagawa 2010)

$$AIC\lambda' = AIC\lambda - 2\sum_{n=1}^{T} \log \left| \frac{dh}{dz} \right|_{z=z_\lambda(n)}$$

dh/dz : Jacobian

$z_\lambda(n) = h^{-1}(y_\lambda(n))$

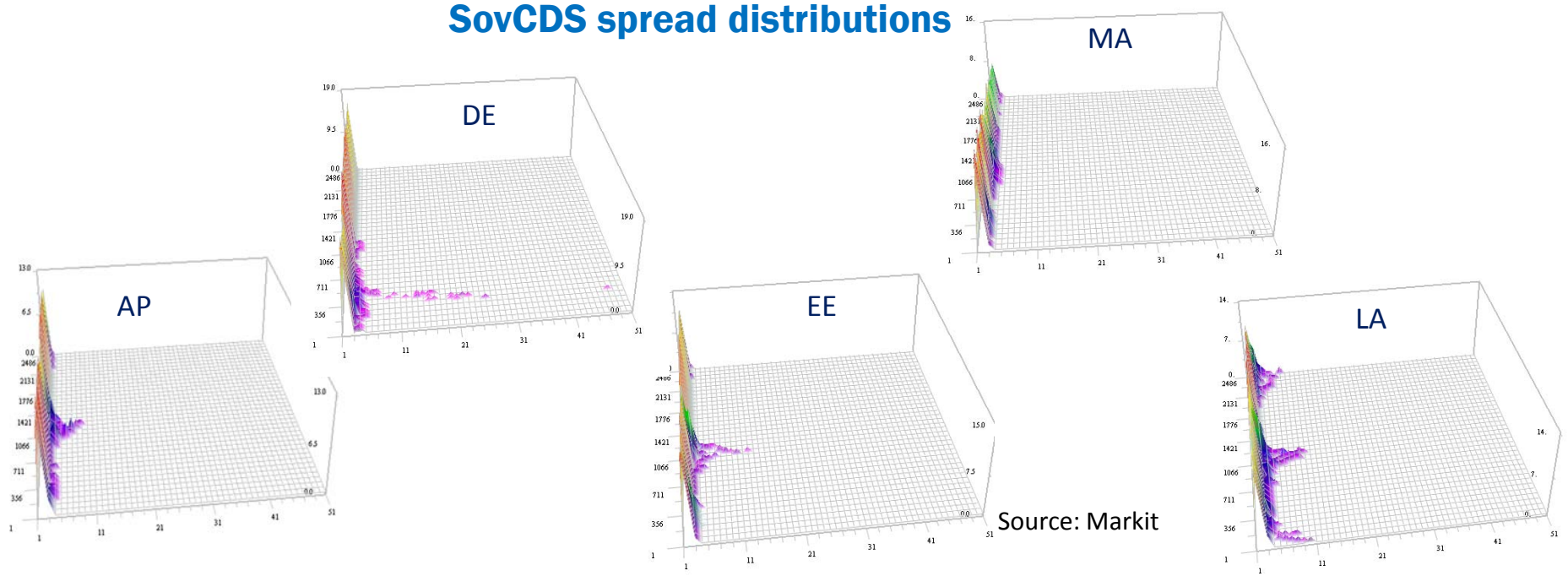
5 An index is defined by the inverse Box-Cox transformation of the optimal trend.

* Improved version of Tanokura et al. 2012

Countries of five regions

Region		Country						
Asia Pacific (AP)	15	Australia	Hong Kong	Japan	New Zealand			
		China	Fiji	Indonesia	Korea	Malaysia		
		Pakistan	Philippines	Sri Lanka	Taiwan	Thailand	Vietnam	
Developed Europe (DE)	19	Denmark	Finland	Iceland	Ireland	Norway	Sweden	UK
		Cyprus	Greece	Italy	Malta	Portugal	Spain	
		Austria	Belgium	France	Germany	Netherlands	Switzerland	
Emerging Europe (EE)	17	Bulgaria	Croatia	Estonia	Hungary	Kazakhstan	Czech Republic	
		Latvia	Lithuania	Macedonia	Poland	Romania	Russia	
		Serbia	Slovakia	Slovenia	Turkey	Ukraine		
Middle East/Africa (MA)	16	Angola	Bahrain	Egypt	Ghana	Iraq	Israel	
		Jordan	Lebanon	Morocco	Nigeria	Oman	Qatar	
		Saudi Arabia	South Africa	Tunisia	UAE			
Latin America (LA)	15	Argentina	Brazil	Chile	Colombia	Costa Rica		
		El Salvador	Guatemala	Jamaica	Mexico	Dominican Rep		
		Panama	Peru	Uruguay	Venezuela	Trinidad & Tobago		
Total	82							

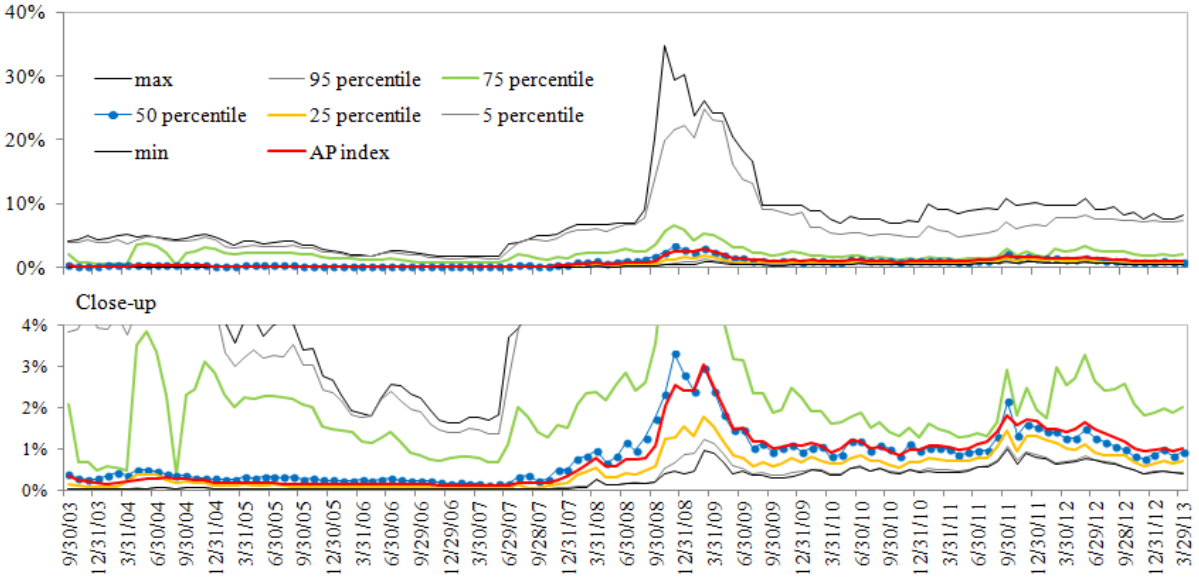
SovCDS spread distributions



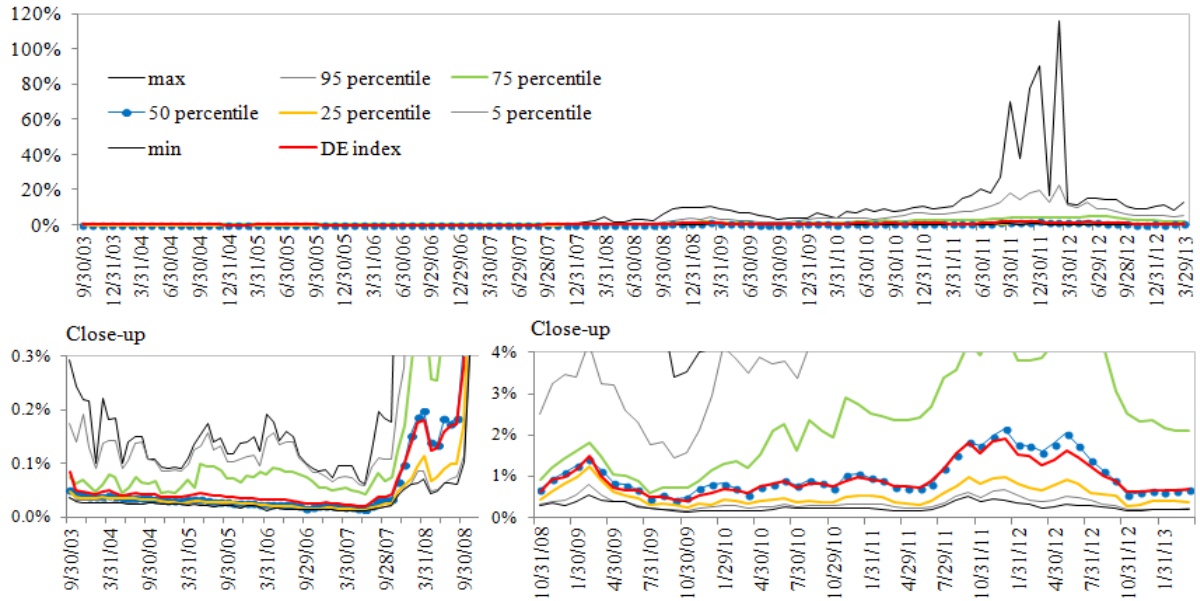
Source: Markit

Sovereign risk index and spread distributions

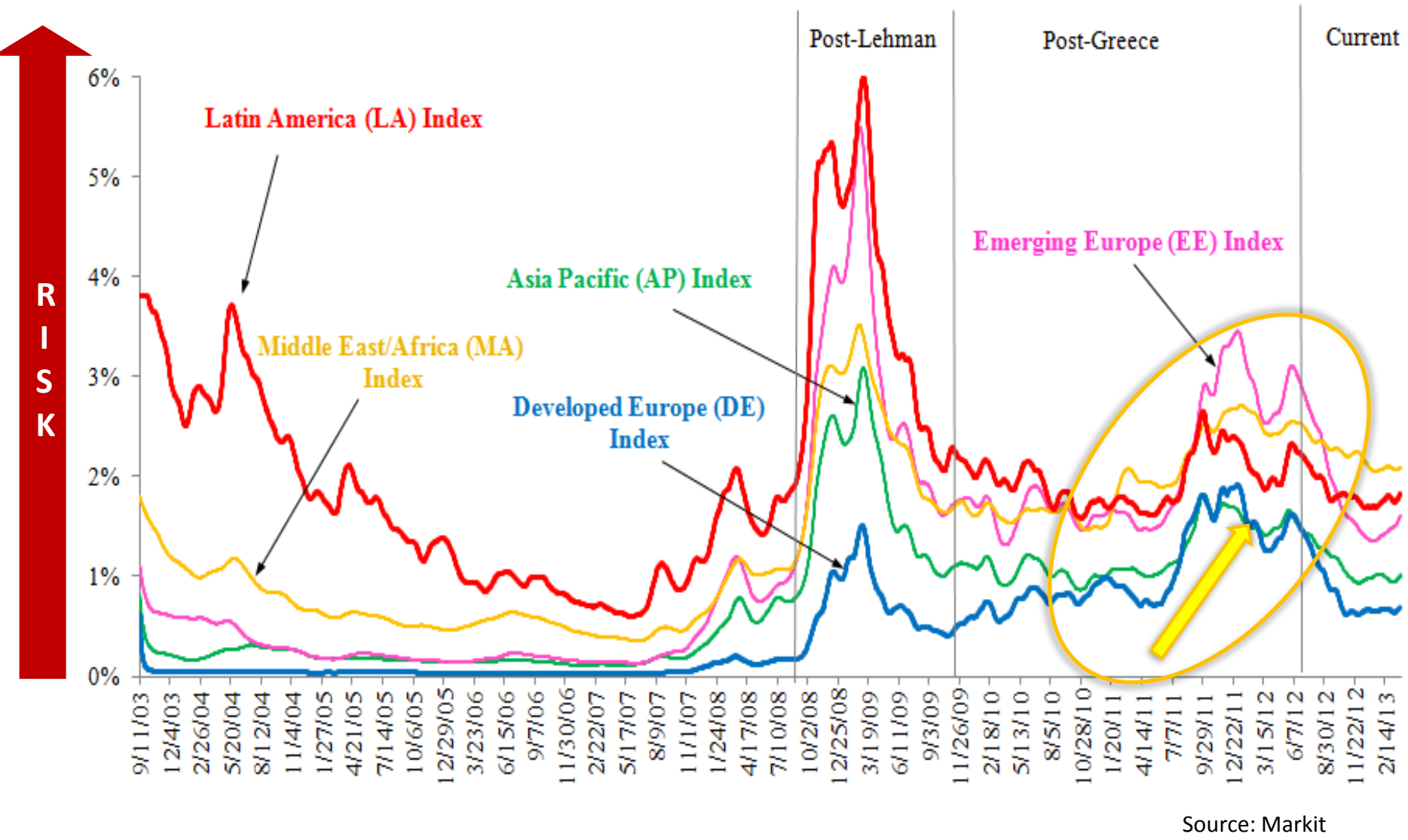
Asia Pacific (AP) Index & spread distributions



Developed Europe (DE) Index & spread distributions



Five regional sovereign risk indices



Source: Markit

Fit a multivariate AutoRegressive model and calculate power contributions(PC)

$$x_n = \sum_{m=1}^{M} A_m x_{n-m} + v_n$$

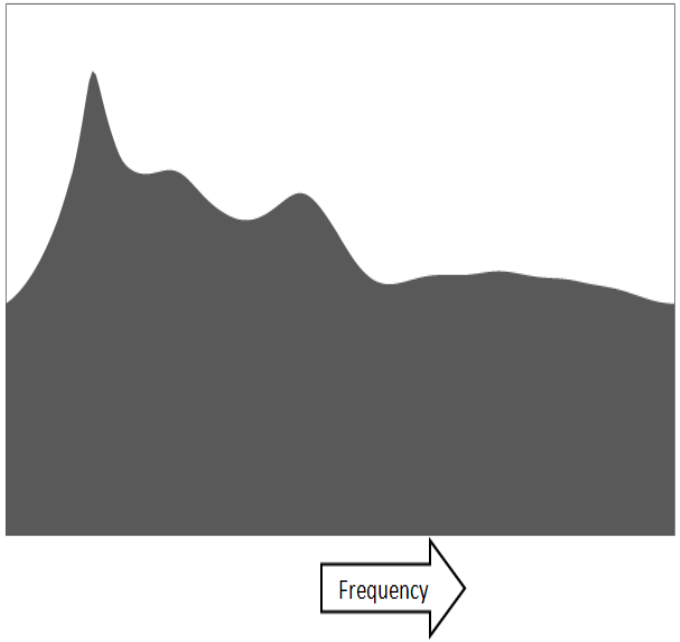
$$E(v_n) = O, \quad E(v_n v_n^T) = W,$$
$$E(v_n v_m^T) = O \ (n \neq m),$$
$$E(v_n x_m^T) = O \ (n > m)$$

x_n : 5-dim stationary time series
A_m : AR coefficient matrix
v_n : 5-dim white noise
W : Variance covariance matrix

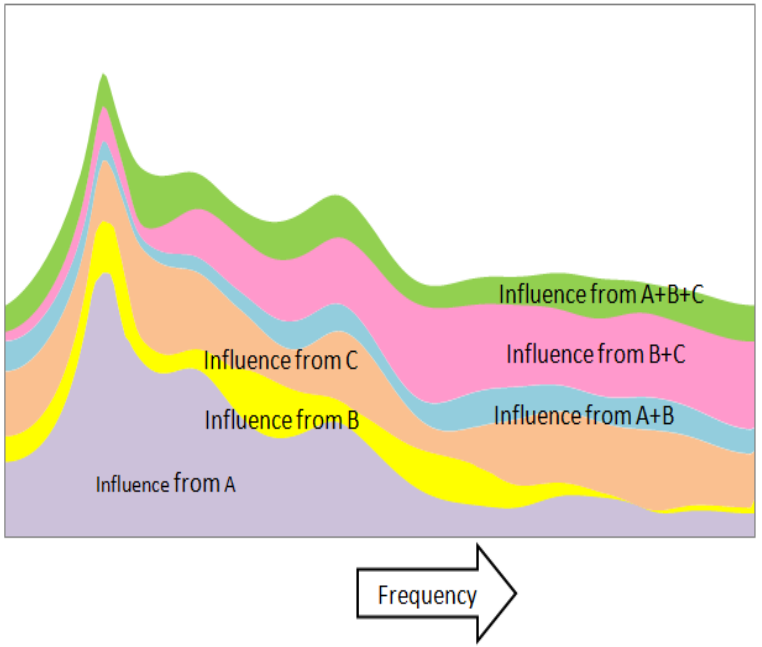
PC measures the influence between variable fluctuations of the noise at a frequency.

(Akaike 1968, Tanokura and Kitagawa 2004)

Power spectrum (PS) of A:
decomposes the fluctuation by frequency

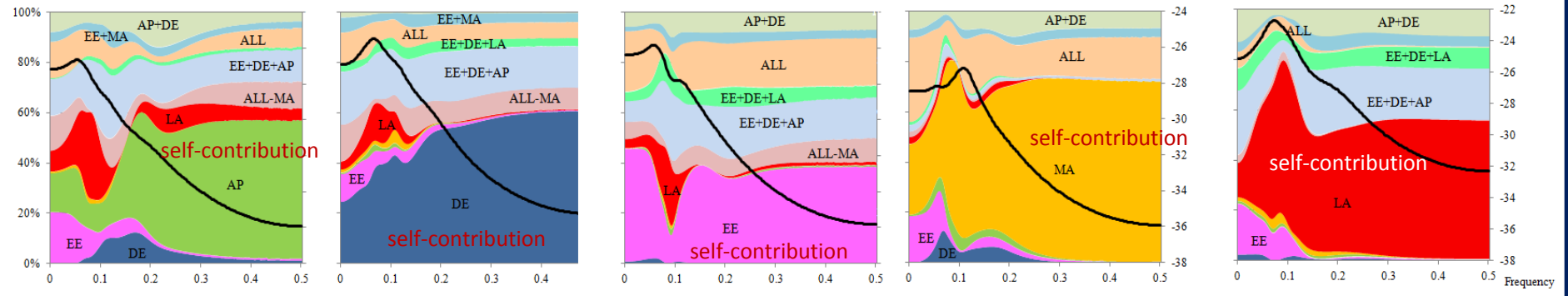


PC of A:
decomposes PS of A into components of variable combinations

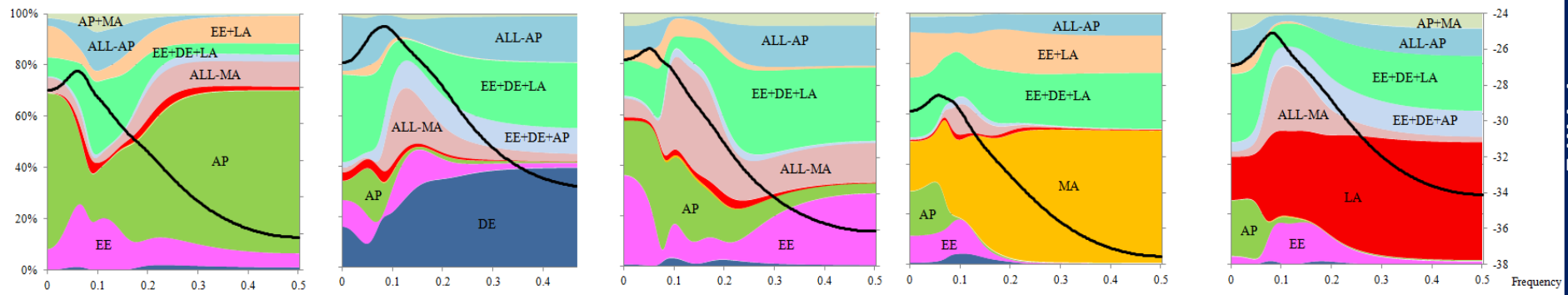


Power contributions (%)

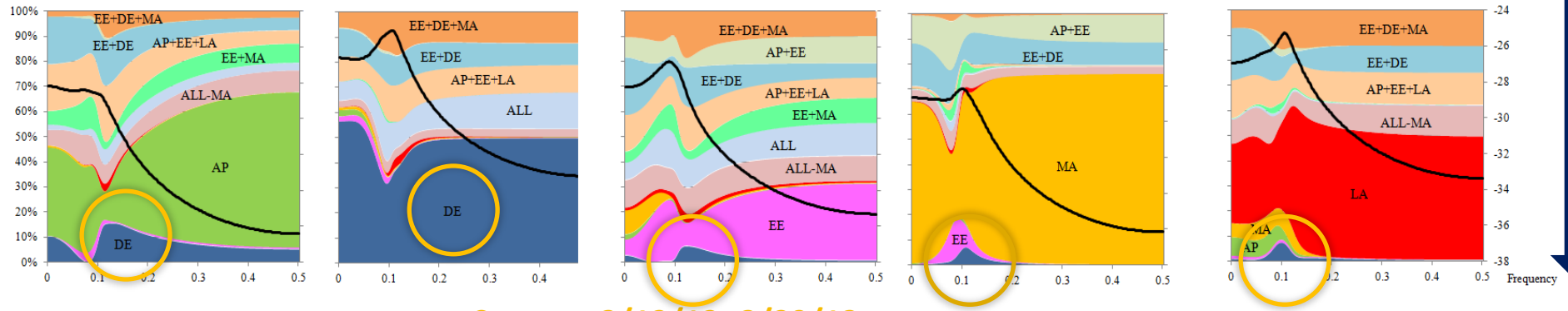
Asia Pacific (AP) Dev. Europe (DE) Emerg. Europe (EE) Mid. East/Africa (MA) Latin America (LA)



Post-Lehman: 9/15/08 -11/16/09



Post-Greece: 11/17/09 -3/8/12

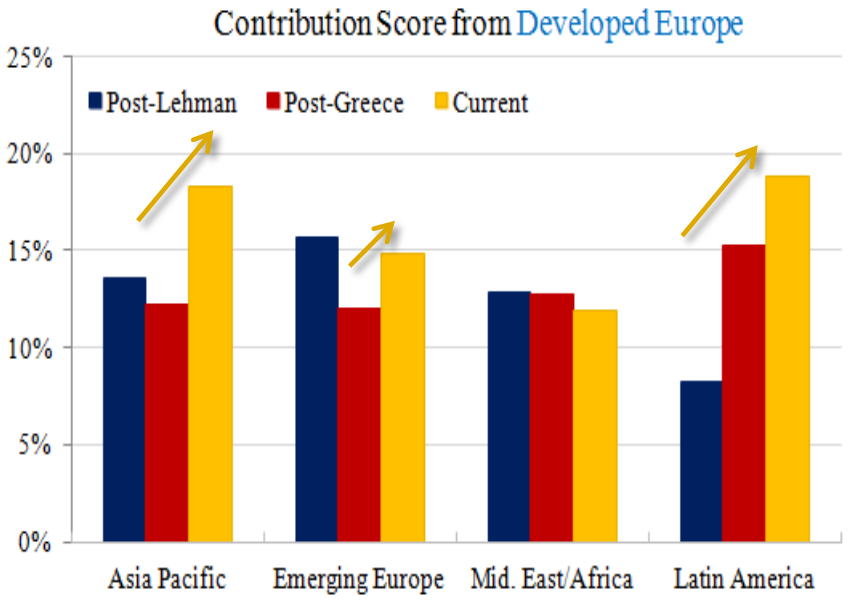


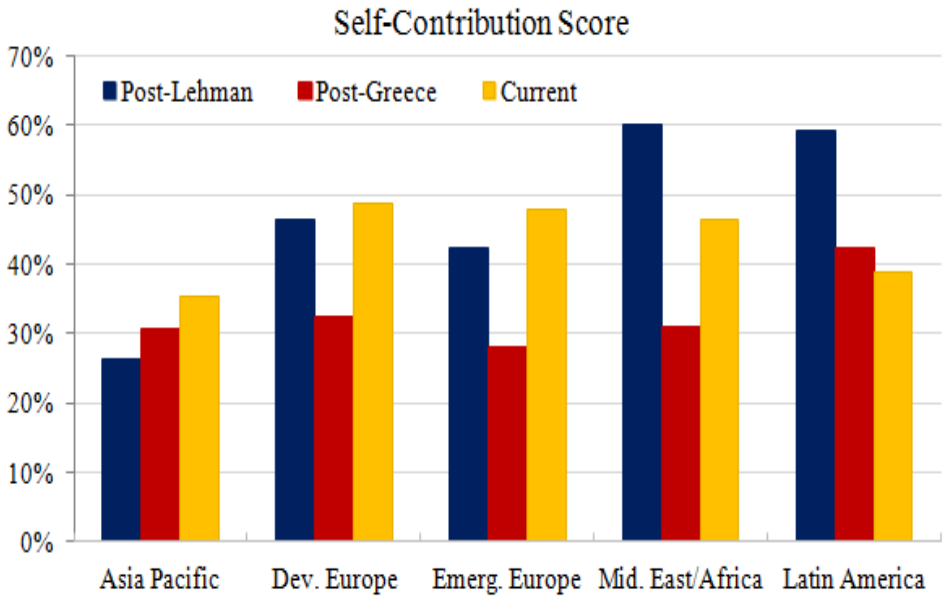
Current: 3/12/12 -3/29/13

TIME

Contribution score (CS)

...is defined as the quantity (% of the total) between two variables based on the sum of the equally allocated the PC value to variables concerned
(at the dominant frequency domain of the power spectrum each region).



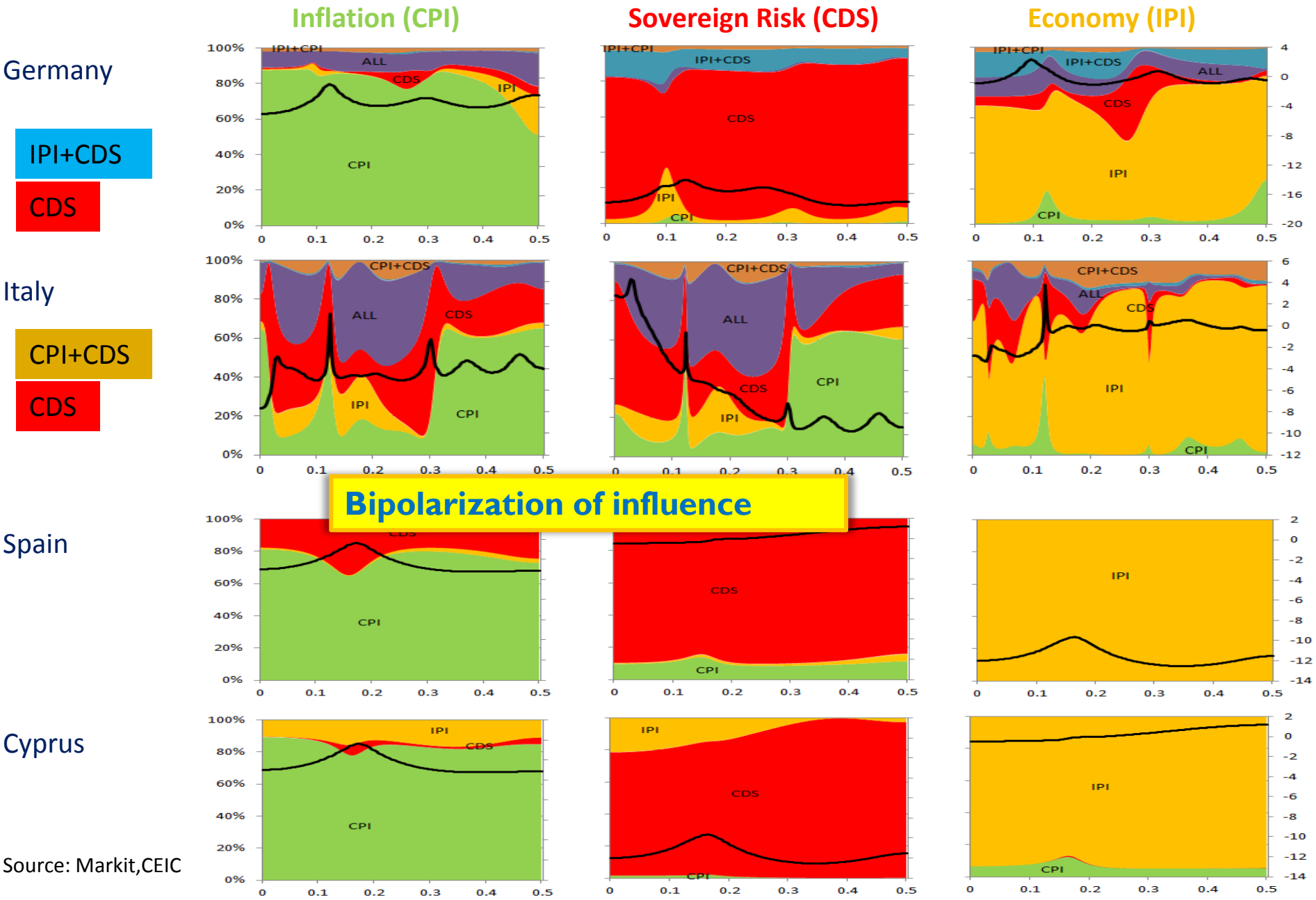


CS from Developed Europe can be regarded as the influence of the European debt crisis.

Worldwide spillover effects are almost found.

CSs from Developed Europe for the current period become higher!

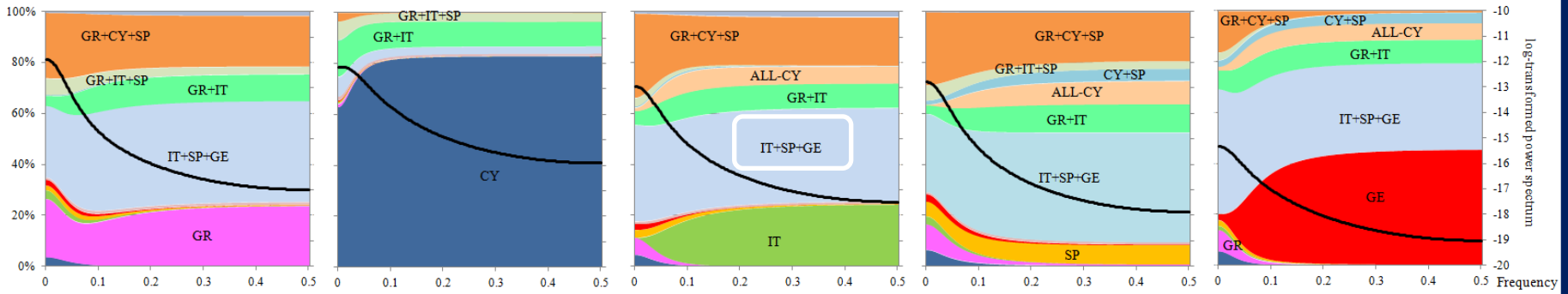
What happened inside the country in Dev. Europe



Source: Markit,CEIC

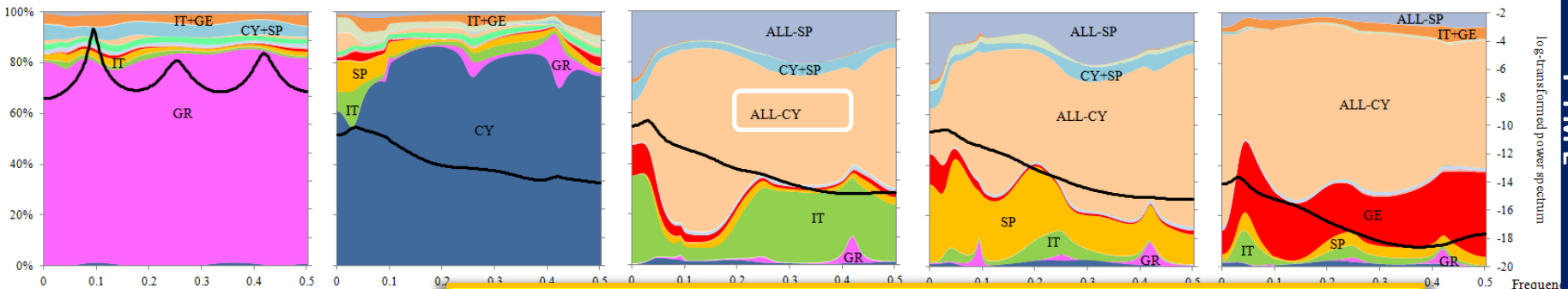
Influence of sovereign risks between countries

Greece (GR) Cyprus (CY) Italy (IT) Spain (SP) Germany (GE)



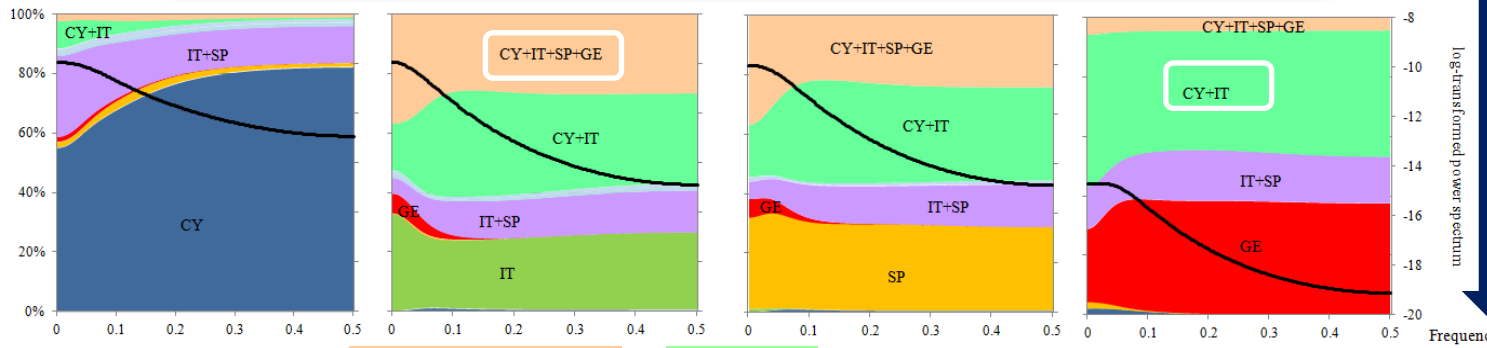
Post-Lehman: 9/15/08 -11/16/09

IT+SP+GE



Post-Greece: 11/17/09 -3/8/12

Departure of Greece and prominence of Cyprus



Current: 3/12/12 -3/29/13

CY+IT+SP+GE

CY+IT

TIME

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