Mathematics : (as a) Mediating Link in Architecture

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Why computational modelling?

- Need to solve a problem, curiosity, inspiration, to learn, to compare, to re-experience.
- Computational models should not be perceived by their parameters and what they represent alone, but also with the process of modelling consisting of several mappings, that is, algorithms constructing the end model.
- They are the mediating links from nature to architecture or
- De-coding and Re-coding in architecture
- AN ARCHITECTURE SOMETIMES WITHOUT AN ARCHITECT!

A groundbreaking paradigmatic change in architecture: computational design

- Particularly after the Information Revolution, architecture has been going through a shift from an interest **in form production towards form generation and form finding**.
- The basic accelerator of such a progress is the intention of deriving an explicit structure of generative design (system), where the architectural product is regarded as the associative outcome of an integrated process that intakes knowledge/ information from various disciplines
- **FROM FORM CENTERED TO PROCESS CENTERED**

Architecture has always been in communication with nature throughout the human history. Some architectural products interacted with nature environmentally, and some internalized nature in the form and generation of design. Common to both, the information existing in natural phenomena has been modelled in a particularity and structure in order to be transferred to architecture. Depending on the complexity and capability of the model, the knowledge extracted and implanted had various inferences for architecture in addition to a variety of scale, proximity, technicality, complexity, and scope.

THE PROCESS-BASED INTELLIGENCE OF NATURAL PHENOMENA IS THE PRIMARY SOURCE OF LEARNING ABOUT SYSTEMS, WHICH CAN INSTRUCT/ CONDUCT ARCHITECTURAL INQUIRY AND THE FORM FINDING EXPERIMENTATIONS/EXPLORATIONS WITHIN THE GENERATIVE PROCESS IT EMBRACES.

Re-visiting nature with computation

CASE 1

From Sea Shell to Man-made Shell

What is shell? Why shell?

- the word shell is commonly used to describe external, usually hard, protective, or enclosing case or covering in nature.
- Similarly, in man-made products the word shell is used to define a rigid covering that envelops an object or a framework or exterior, as of a building.
- Shell structures are greatly superior to conventional column-beam structures when seeking to cover large spans with minimum material. spans, carrying capacity compromising live and dead loads is very efficient in shells.
- Although shells have the most complex structural behaviors they are still the most effective form.

Decoding and Re-coding Seashell

- A shell is a three dimensional curved structure which resists load through its inherent curvature.
- There are numerous shell structures in nature; eggs, skulls, nuts, turtles and seashells, are notable examples
- A shell’s structural behaviour is derived directly from its form,
- This not only dictates the æsthetics, but the overall efficiency and behaviour under load of the structural system.

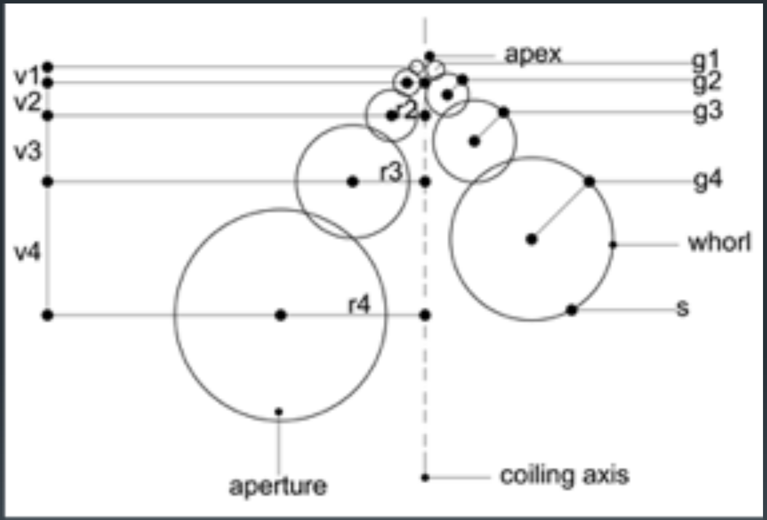
First parameters…and a new parametric model

S: Section; Shape of the aperture or shape of the shell's tube cross section

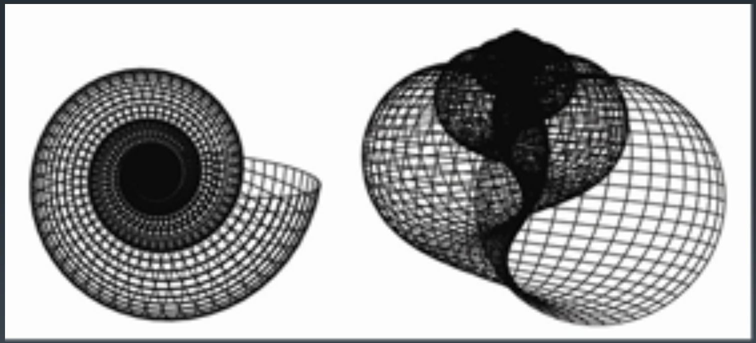
H: Horizontal Displacement; Departure from the coiling axis of the section in the horizontal direction

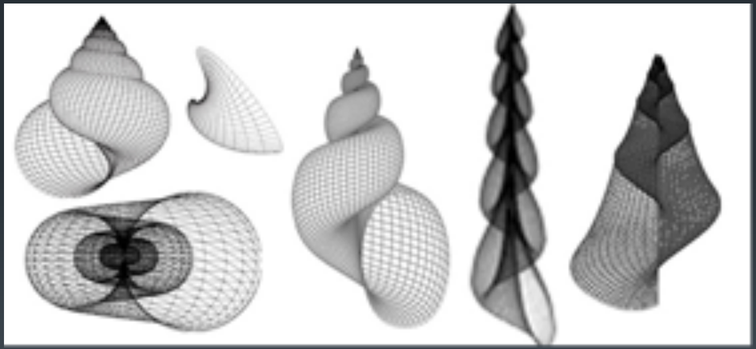
V: Vertical Displacement; Translation along the vertical direction of the coiling axis

G: Growth: Aperture expansion or the rate of increase of section size



Abstracted simple seashell models generated by the initial model ….





We should understand more!!! Seashell growth…

Seashells are formed in nature by growth at the shell's free leading edge.

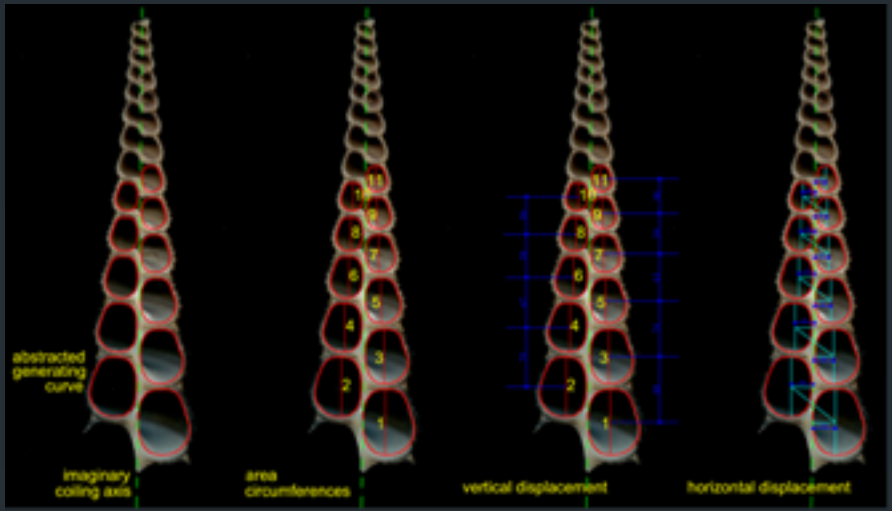
Their increase in overall size is achieved purely from successive addition of material to one end only [

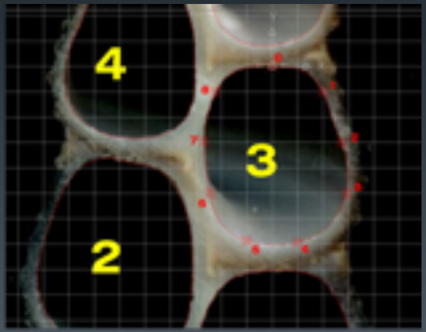
From inspection of actual seashell cross-sections, the older previously formed parts of the shell remain, on the whole, unaffected and geometrically unchanged once produced

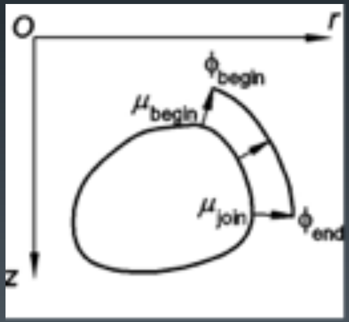
Cut cross-sections of a selection of seashells found in natüre (METU)

Let us focus more!

Simple models are not good enough to understand structure-form relation







Be more precise!

B-Splines to model seashells…

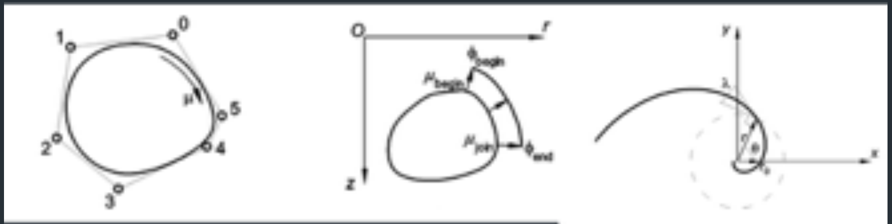
A parametric model of the shell was set up using the cross section of a single whorl as the input. In a similar approach to Fowler et al [the shape of a single typical whorl was defined as a B-spline

The shell surface geometry is defined using cylindrical polar coordinates

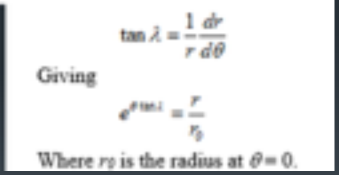
X=rcosΘ

y=rsinΘ

Z=z



B-spline and its control polygon (right) single whorl cross-section

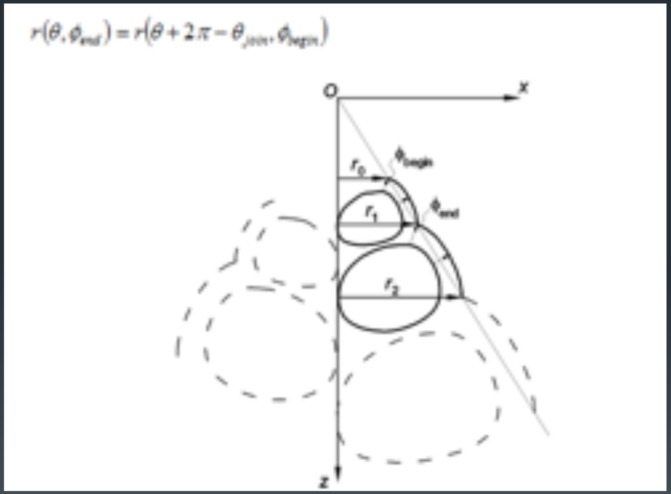


Geometry of the logarithmic spiral.

Towards a realistic model…

From observations made from real seashells the growth rings, which correspond to the whorl cross-section, are not radial and do not even lie in one plane.

They are curves in three dimensions. This means that the shell's rate of spiral, λ are ϕ_{begin} on the current leading edge cross-section must lie coincident with ϕ_{end} on the preceding section after slightly less than one revolution about the major z-axis, i.e. $\Delta\theta = 2\pi - \theta_{join}$,



Geometric relations of the shell surface

New model…

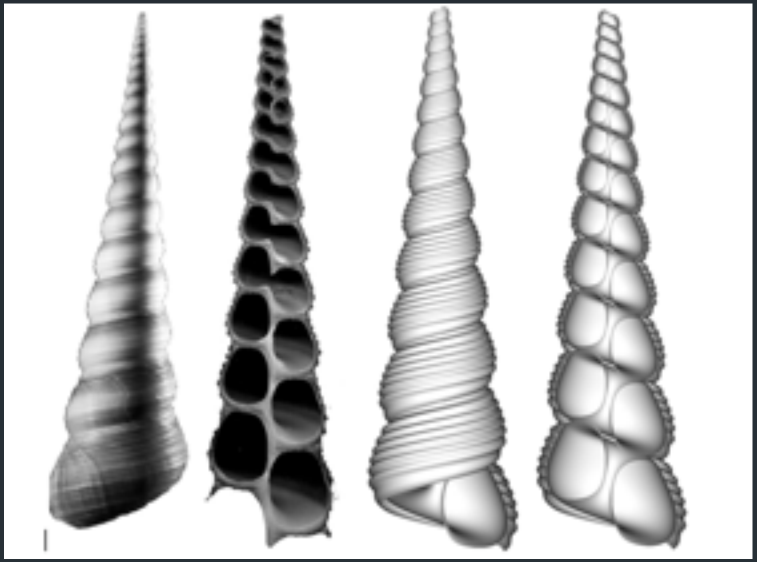
Thus λ is controlled by the relationship:

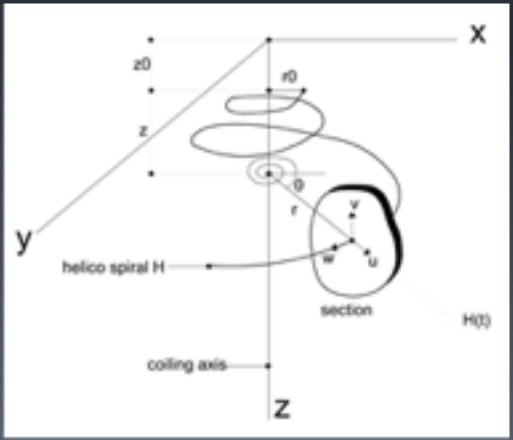
$$e^{(2\pi-\theta_{join})\tan\lambda} = \frac{r_1}{r_0}$$

λ = rate of spiral constant
r_0 = radius to point ϕ_0
r_1 = radius to point ϕ_0'
$\theta_{join} - \theta_{half}$ at point ϕ_{join}

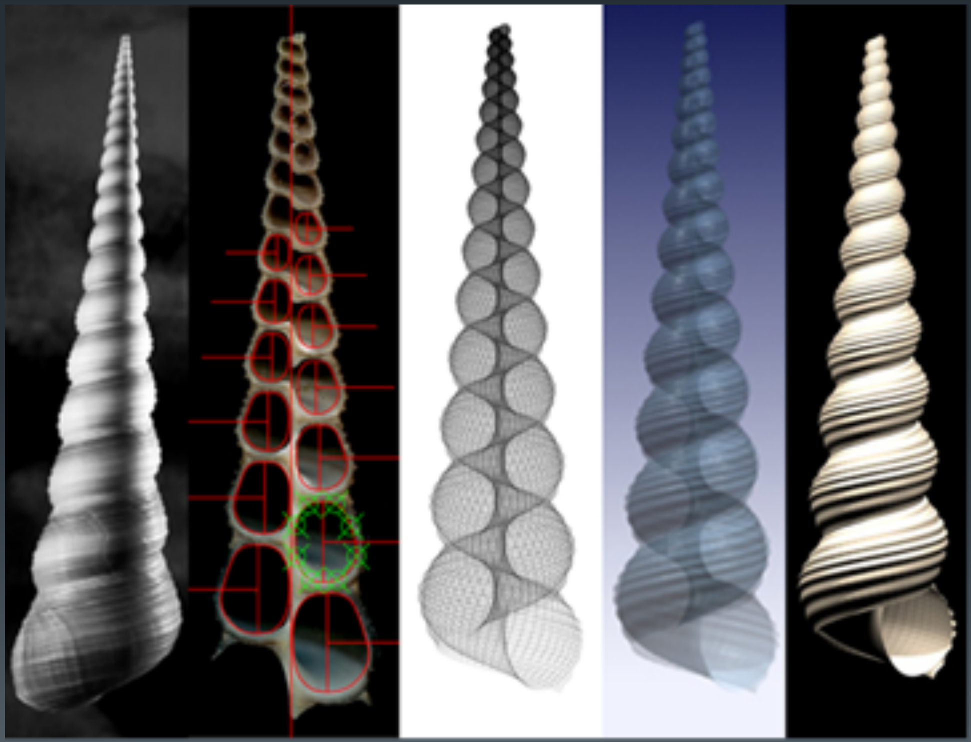
The growth constant is applied to the whorl by transforming its coordinates using the formulae below, based on the cylindrical polar coordinate transformation

$$x = e^{\theta\tan\lambda} r\cos\theta$$
$$y = e^{\theta\tan\lambda} r\sin\theta$$
$$z = e^{\theta\tan\lambda} z$$

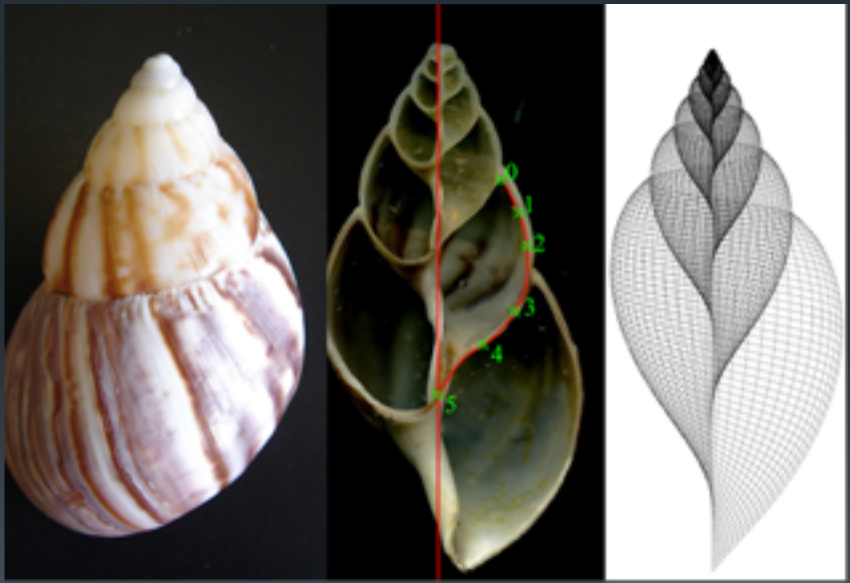




Scan of the seashell found in nature (left) computer generated shell model (right) by the algorithm developed for this research.



The model is generative enough…

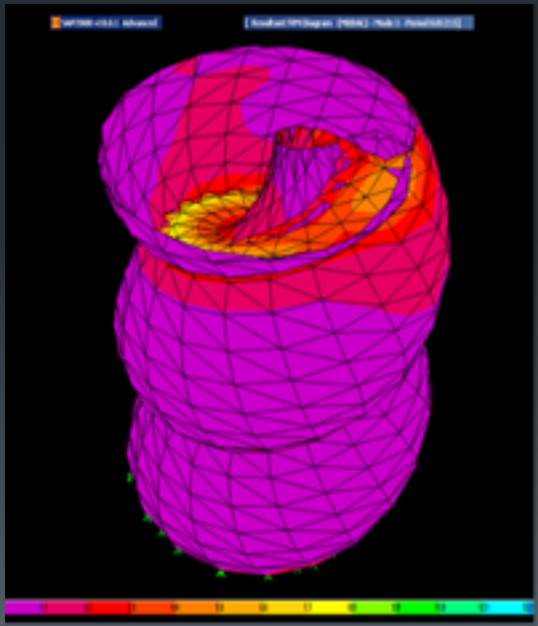


Demonstration of the flexibility of the program to experience different coiled seashell forms.

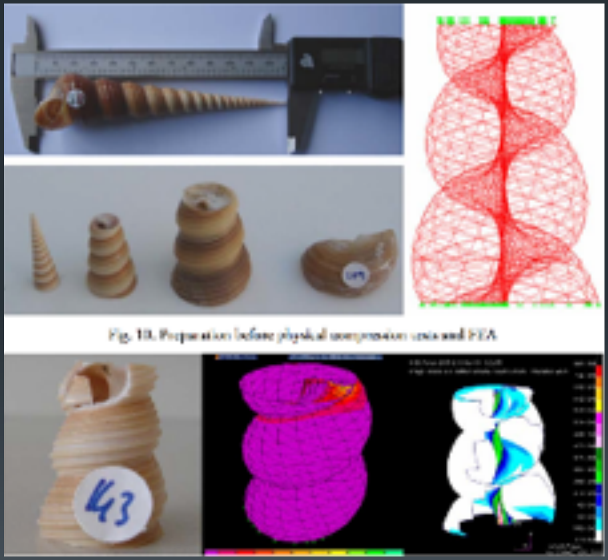
Stress-Form relations

Compression tests in lab

FEM results



Some Non-dimensional Parameters to Represent a Shell



base/height ratio	0.35-0.38	0.40-0.48	0.50-0.59	0.60-0.69	0.70-0.76	0.79-0.86	0.90-1.00
load (N)	543	548	628	649	662	869	931

weight (gr)	7.54-8.18	8.48-9.20	9.50-10.89	12.12-14.00	15.38-16.03	17.18-20.08
load (N)	380	450	820	670	750	820

whorl number	1.5-2	2.5-3	3.5-4	4.5-5	5.5-6	7.5-8	9
load (N)	500	540	560	550	570	620	630

What we learnt?

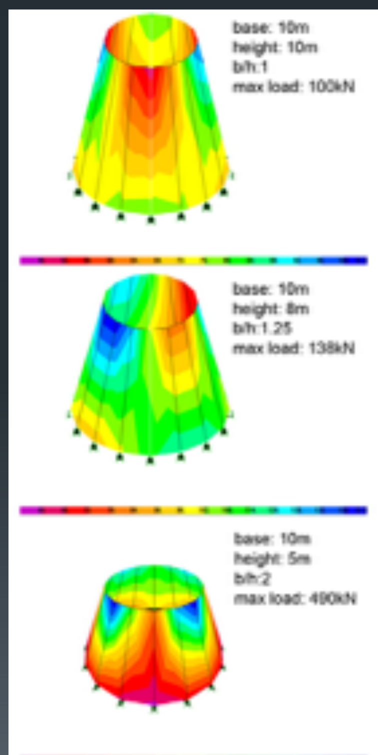


Table 5.15 Diameter to height ratios of some important dome structures (Source: http://archnet.org)

the building	construction year	diameter/ height ratio
Ayasofya	537	2.02
Mahmut Pasa Mosque	1464	1.69
Fatih Mosque	1470	2.01
Sultan Beyazid Mosque	1486	1.31
Sehzade Mosque	1548	1.63
Suleymaniye Mosque	1557	1.64
Kara Ahmet Pasa Mosque	1558	1.64
Rustem Pasa Mosque	1561	1.84
Mihrimah Sultan Mosque	1565	1.54
Lala Mustafa Pasa Mosque	1565	2.03
Selimiye Mosque	1574	1.97
Sokullu Mehmet Pasa Mosque	1577	1.62
Azapkapi Mosque	1578	1.74
Kilic Ali Pasa Mosque	1580	1.50
Meclis Mosque	1987	1.97
Ataevler Merkez Edebali Mosque (the biggest domed mosque in the world)	2008	1.36

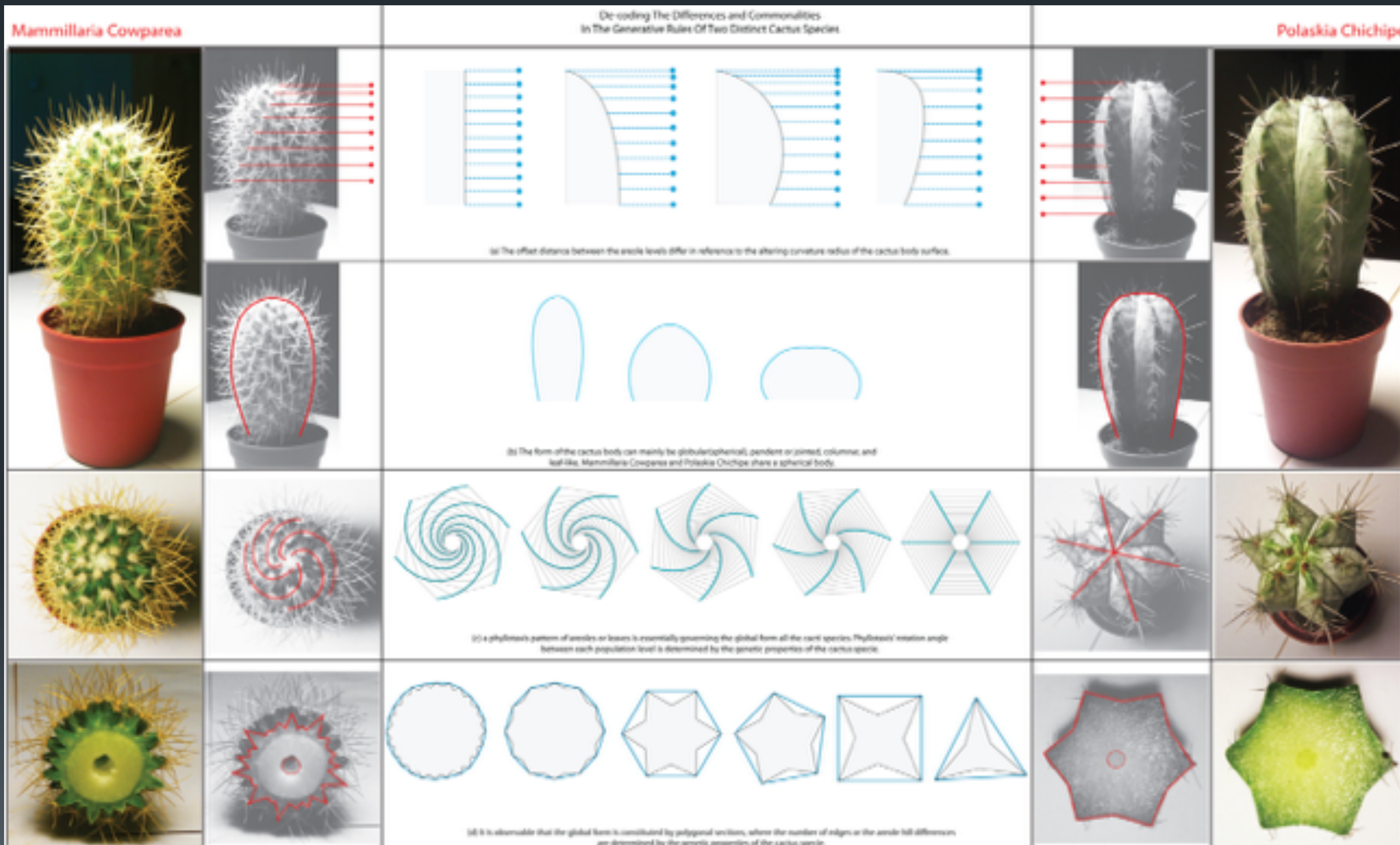
CASE 2

Cactus to highrise….

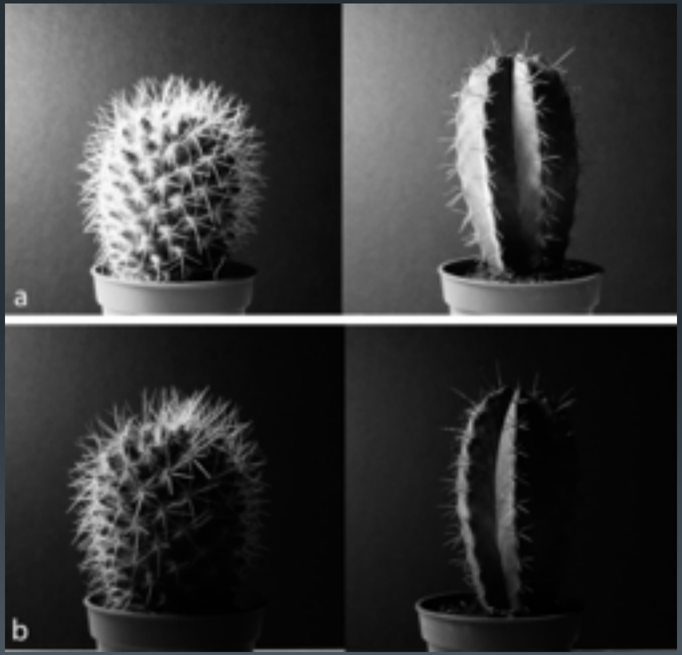
What we will learn from cactus?

- “Thus, the architects wishing to use this new tool[genetic algorithms] must not only become hackers(so that they can create the code needed to bring extensive and intensive aspects together) but also be able “to hack”biology, thermodynamics, mathematics, and other areas of science to tapinto necessary resources.”

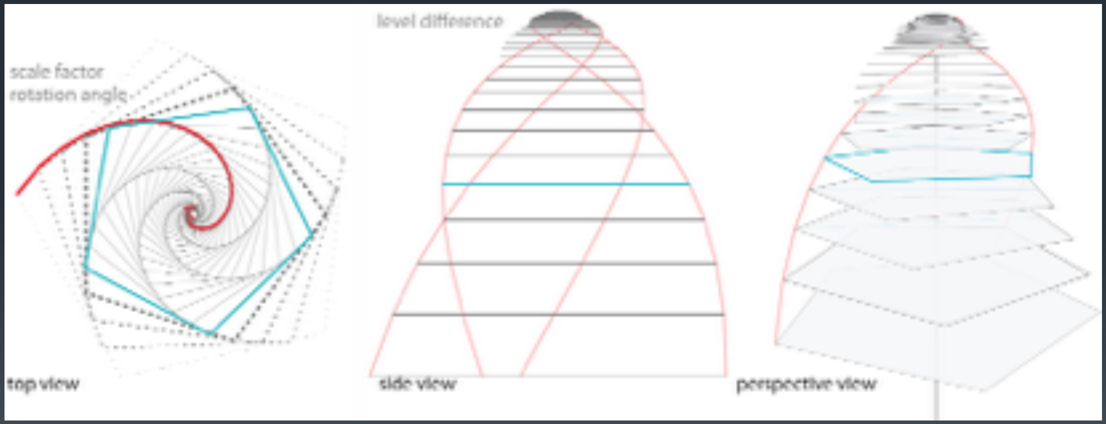
Manuel DeLanda. “Delueze and the Use of the Genetic Algorithm in Architecture.” in Phylogenesis: foa’s ark. Edited by Foreign Office Architects. Barcelona: Actar, 2004. p.529.


Mammillaria Cowparea
De-coding The Differences and Commonalities
In The Generative Rules Of Two Distinct Cactus Species
Polaskia Chichipe
(a) The offset distance between the areole levels differ in reference to the altering curvature radius of the cactus body surface.
(b) The form of the cactus body can mainly be globular(spherical), pendent or jointed, columnar, and leaf-like. Mammillaria Cowparea and Polaskia Chichipe share a spherical body.

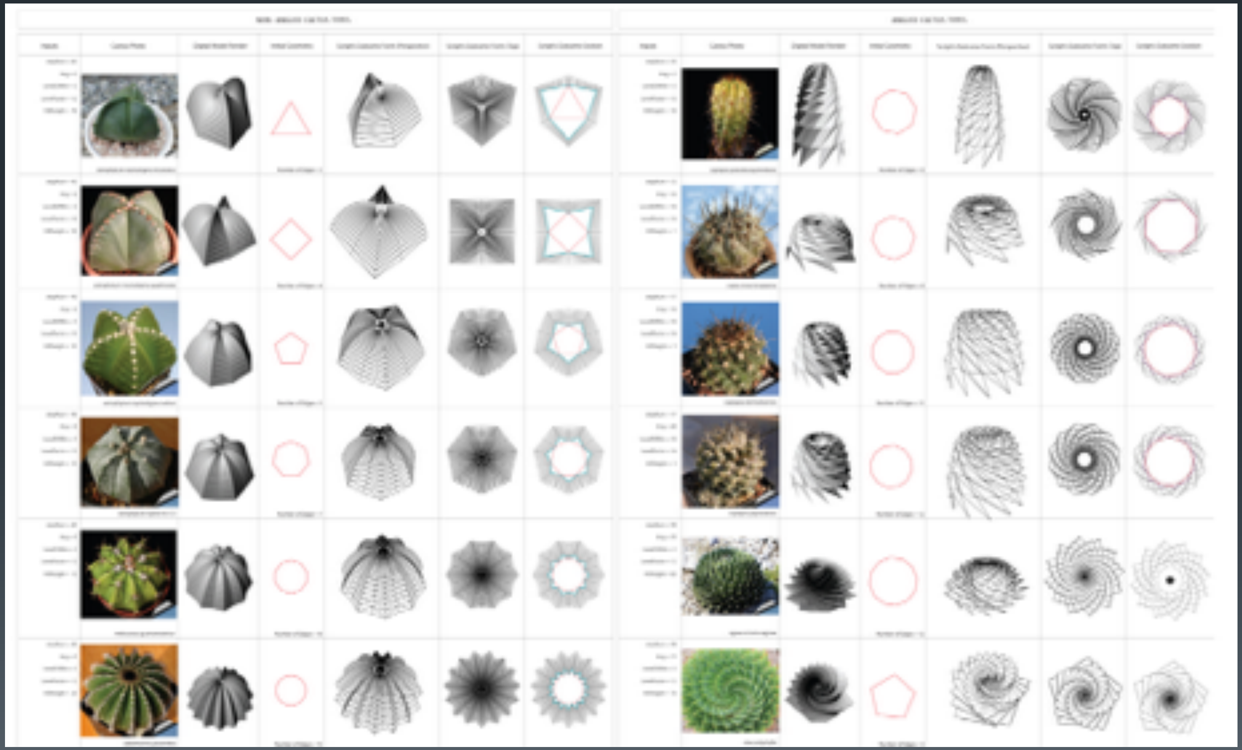
Self-shadowing, self-ventilation



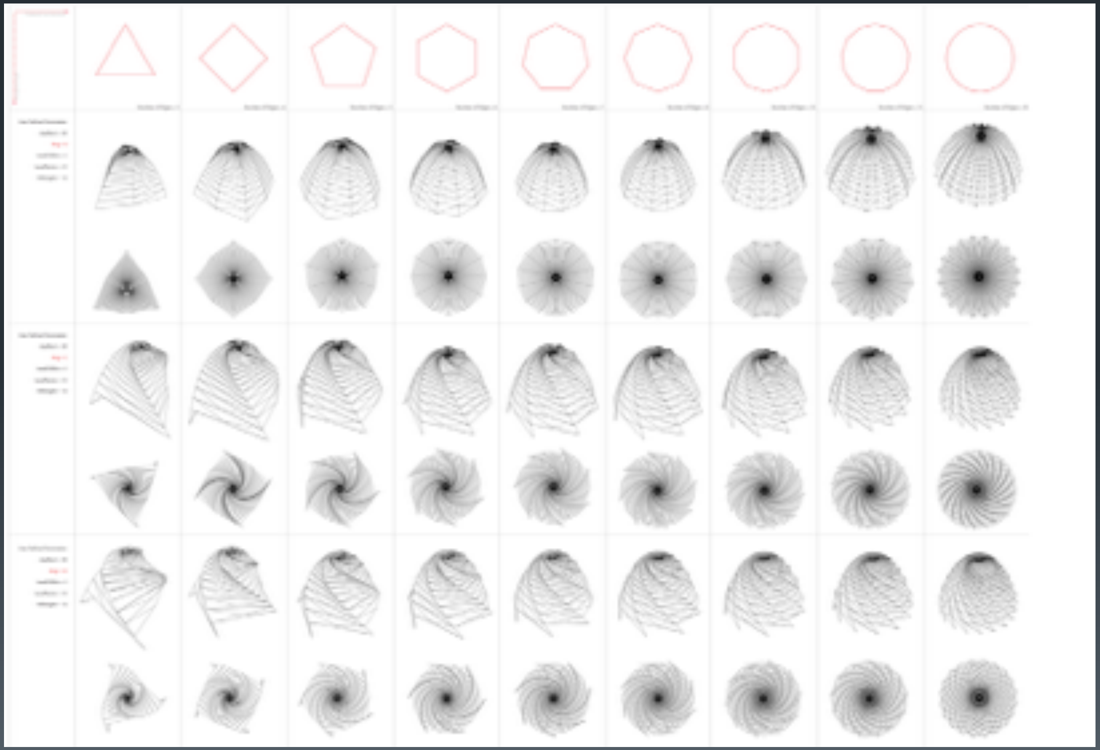
The ‘shared body plan’ of the cactus family. A schematic representation of parameters, operations, relations and the generative set-up at a high level of abstraction



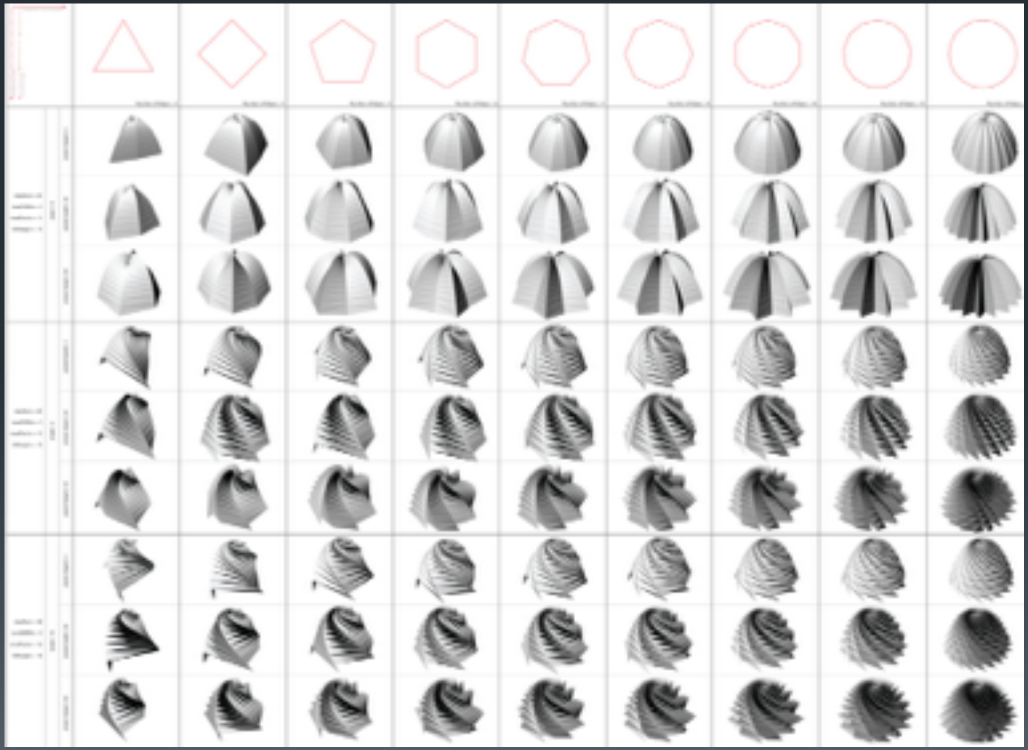
Testing the Correspondence of the Script Outcomes With The Cacti Species.



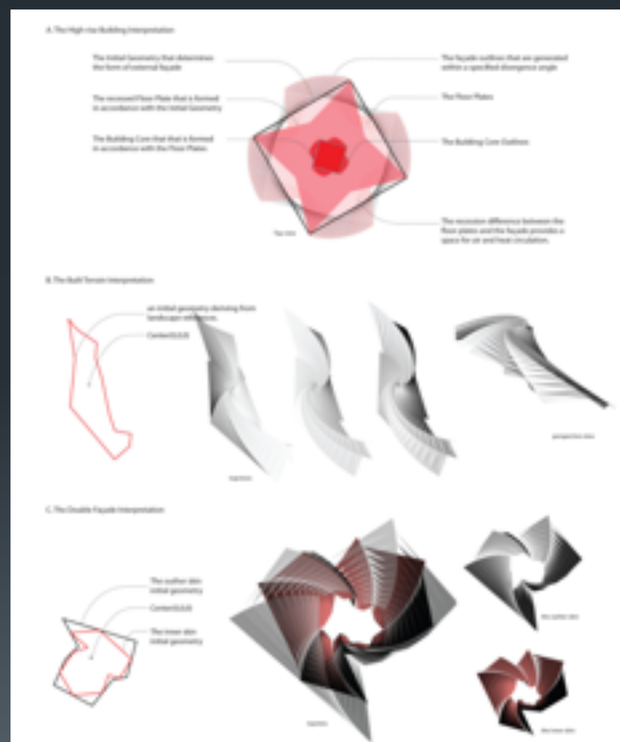
Populations of the Script With Different Initial Polygonal Geometries, and Different Divergence Angles



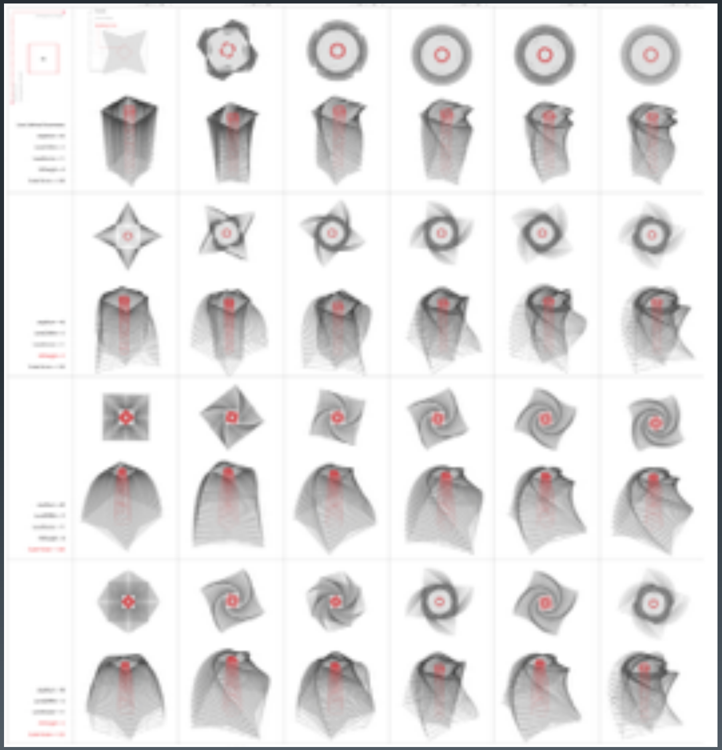
Populations of the Script With Different Areole Hill Depths



Three different architectural projections of the Cactus Script: (a) High-rise buildings, (b) built terrains, (c) double façades.



A Set of Possible Architectural Inferences of the Script for High-rise Buildings.

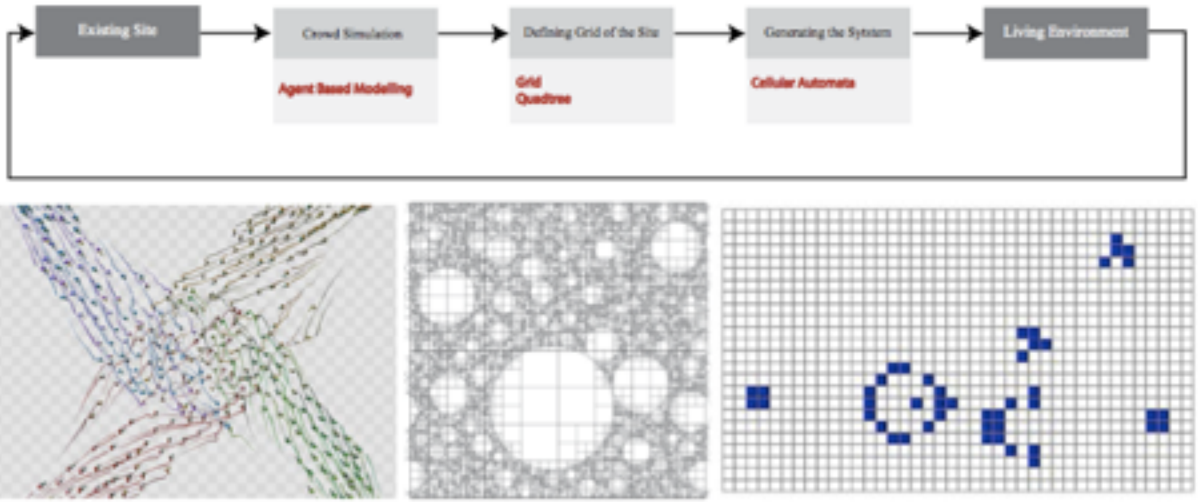


Case 3: Human Crowd Simulation

Muge Krusa M.Sc Supervisor A.Gonenc Sorguc

Case 4: Human Crowd Simulation

Proposed model aims providing organization schemas which reflects the relation between crowd behavior and site organization. Hence, **Cellular Automata** and **Agent Based Modelling** has been implemented together with customization in the algorithms. The connection between the algorithm has been constructed with **quad-tree** algorithm



Basic Assumptions

The model has been developed based on the following **assumptions**.

Assumption regarding **Site**:

- The site should be well defined and **boundaries** should be known.
- The site is presumed as **two dimensional** and any topographical change has been ignored.
- It is assumed that each site has at least one direction for pedestrian movement.(**major movement axis**)

Assumptions about **Crowd Behavior**

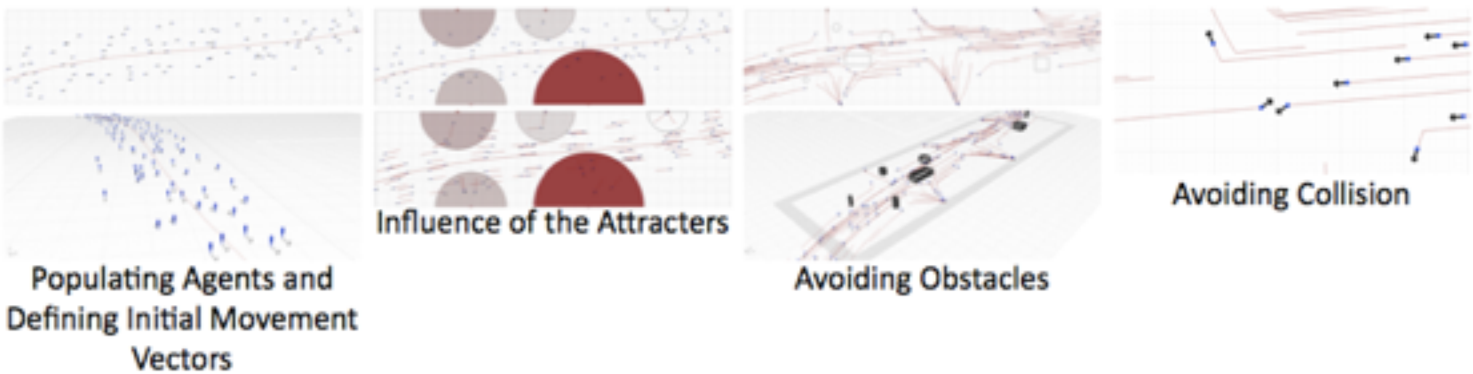
- People experiencing the site are defined as agents with reference to swarm behavior, and they are simplified as points in the model.
- Decisions regarding possible population of agents, their initial distribution on the site, and their density are made by the designer in the model.
- Natural directions of agents are assumed to be dependent on major movement axis. The rest of the crowd will diverge from majority as being attracted by crowd-pulling components such as entrance of buildings and event locations.
- Also, swarm behavior aspects like obstacle and collision avoidance for agents are included in the algorithm.

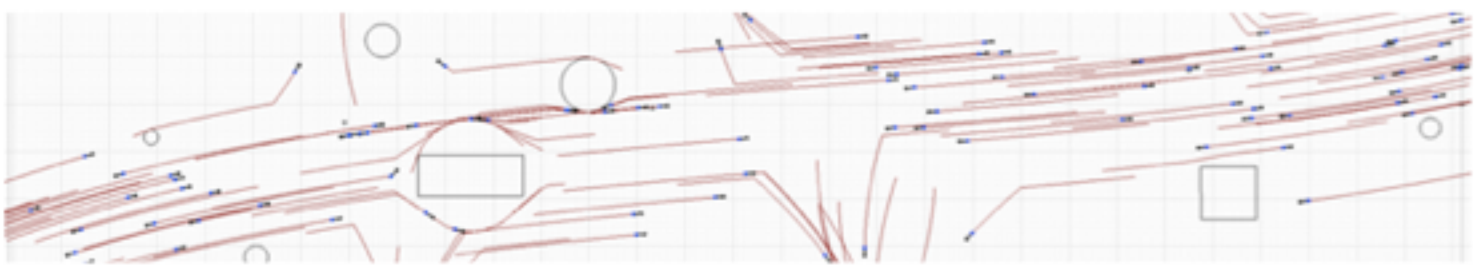
Assumtions considering **Cellular Automaton algorithm**

- Instead of using conventional grid system, the site has been divided into nonstandard grids produced by quad tree algorithm and movement paths.

Crowd Simulation Algorithm

In order to construct crowd behavior model for a part of a space design tool, following emergence and swarm behavior aspects has been analyzed, reinterpreted and remodeled.

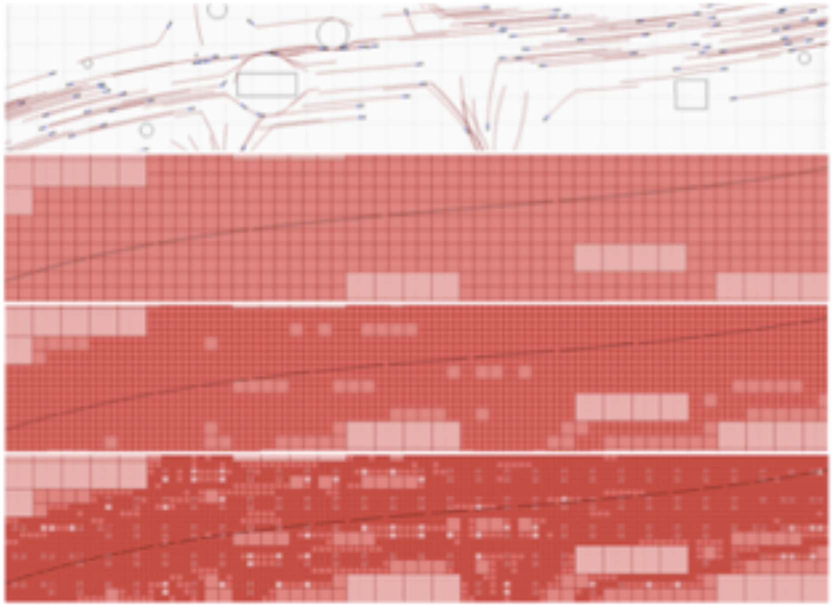


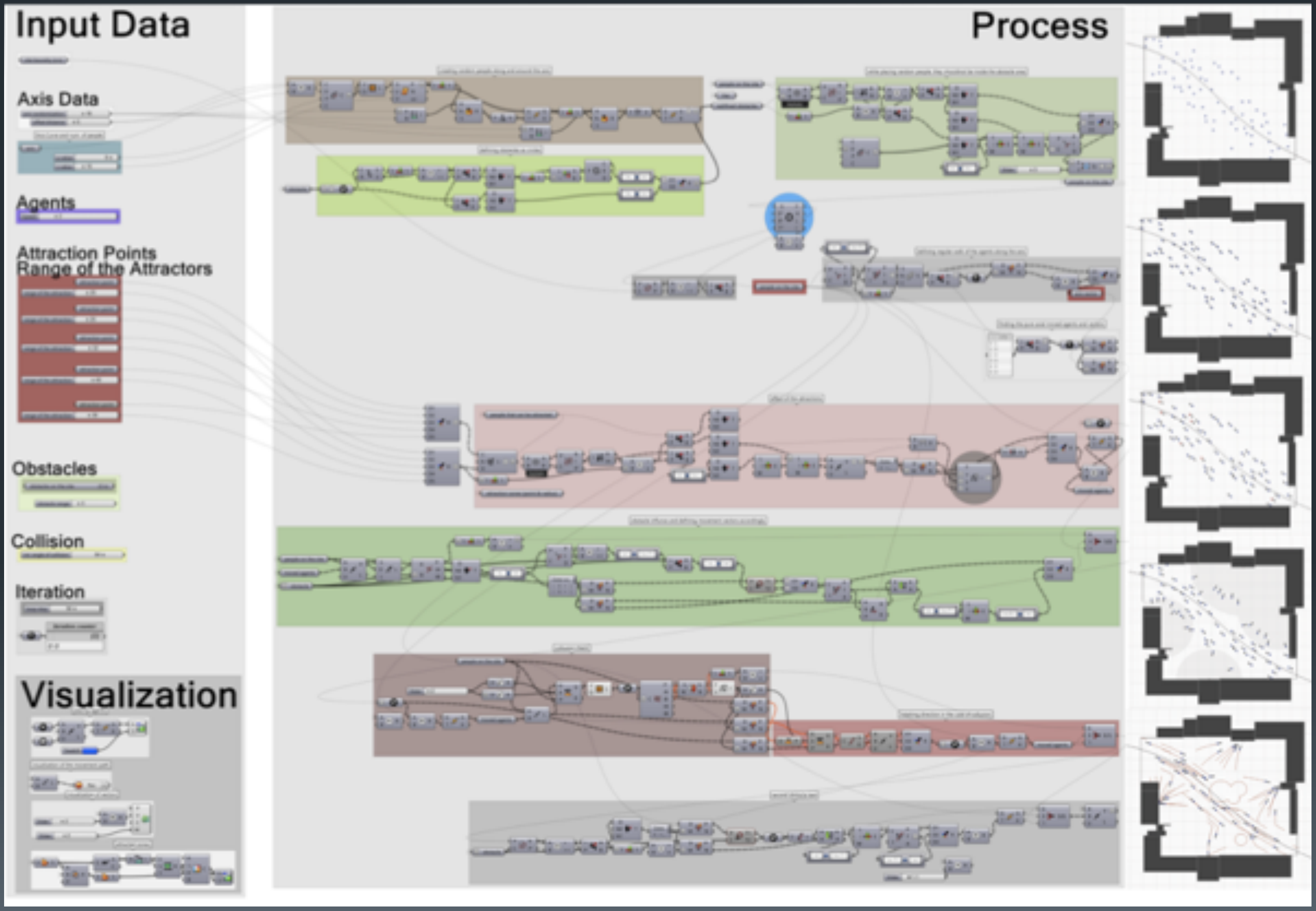


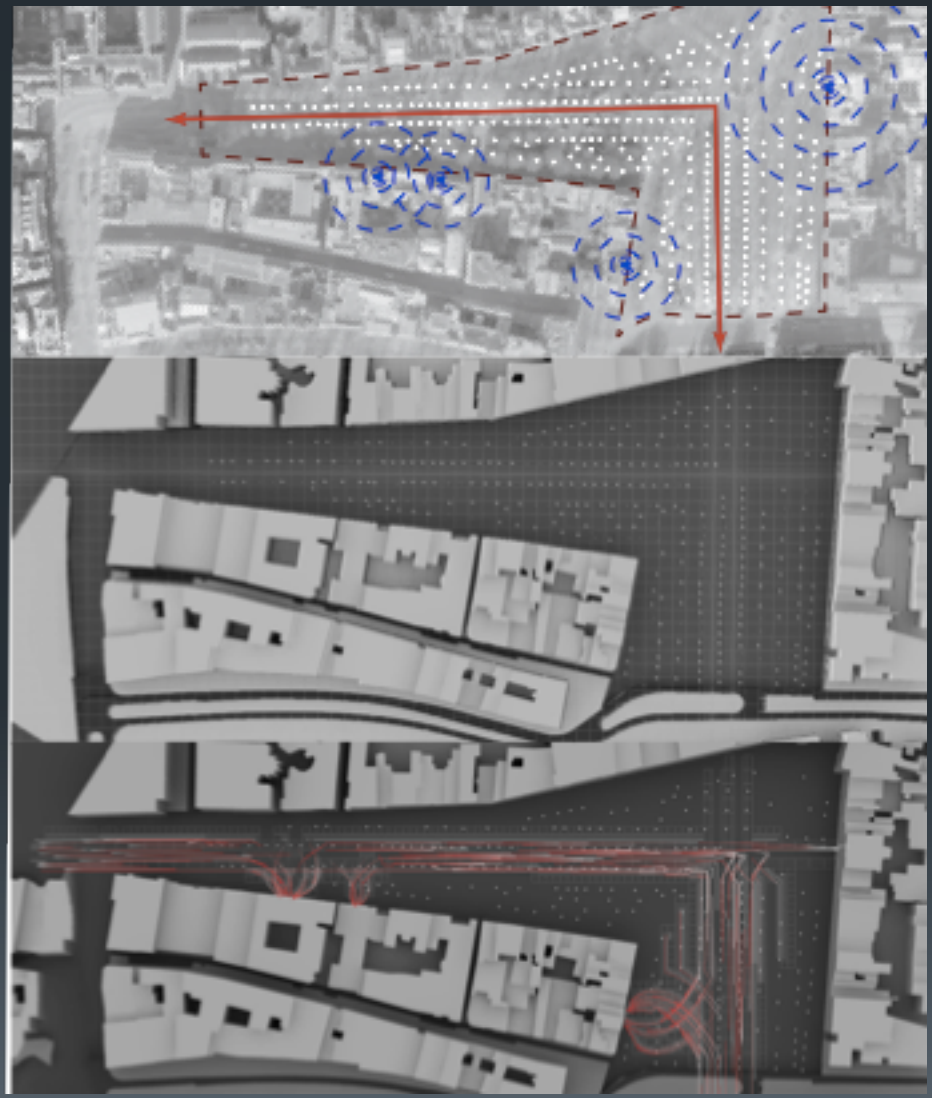
Quad-tree Algorithm and Cellular Automata

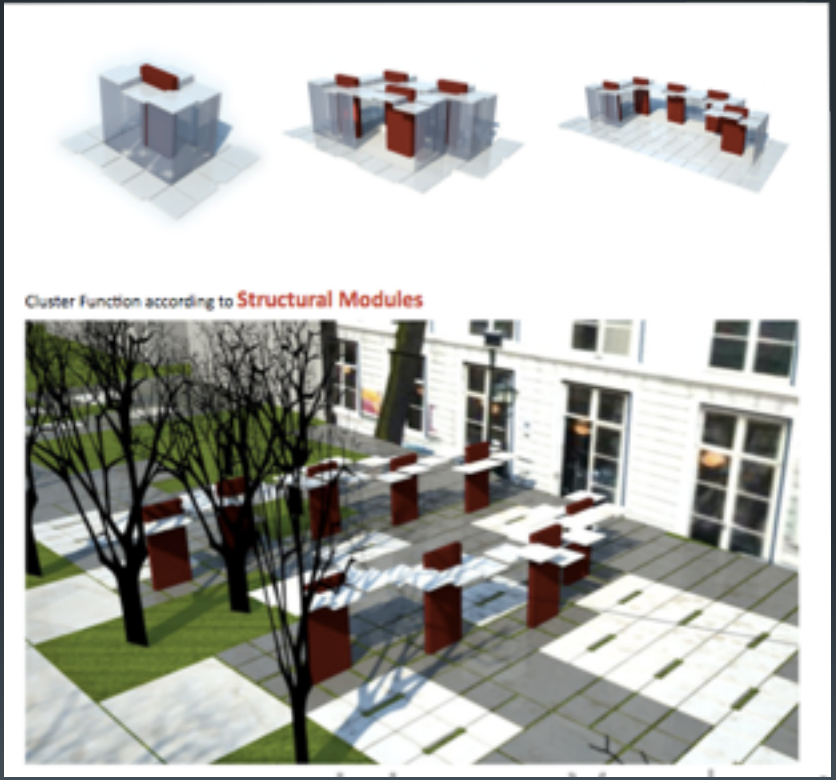
Cellular Automaton model is embraced and generated with non-uniform grid based on quadtree algorithm as the model has been developed for multipurpose open-air settlement regarding adaptability.

Initial square grid is adapted according to agents' movement paths on the site. During this process, retrieved movement paths and user defined initial grid has been placed and their relations are calculated based on quad-tree algorithm.




Input Data
Process
Axis Data
Agents
Attraction Points
Range of the Attractors
Obstacles
Collision
Iteration
Visualization




Cluster Function according to Structural Modules